Channel Transitions

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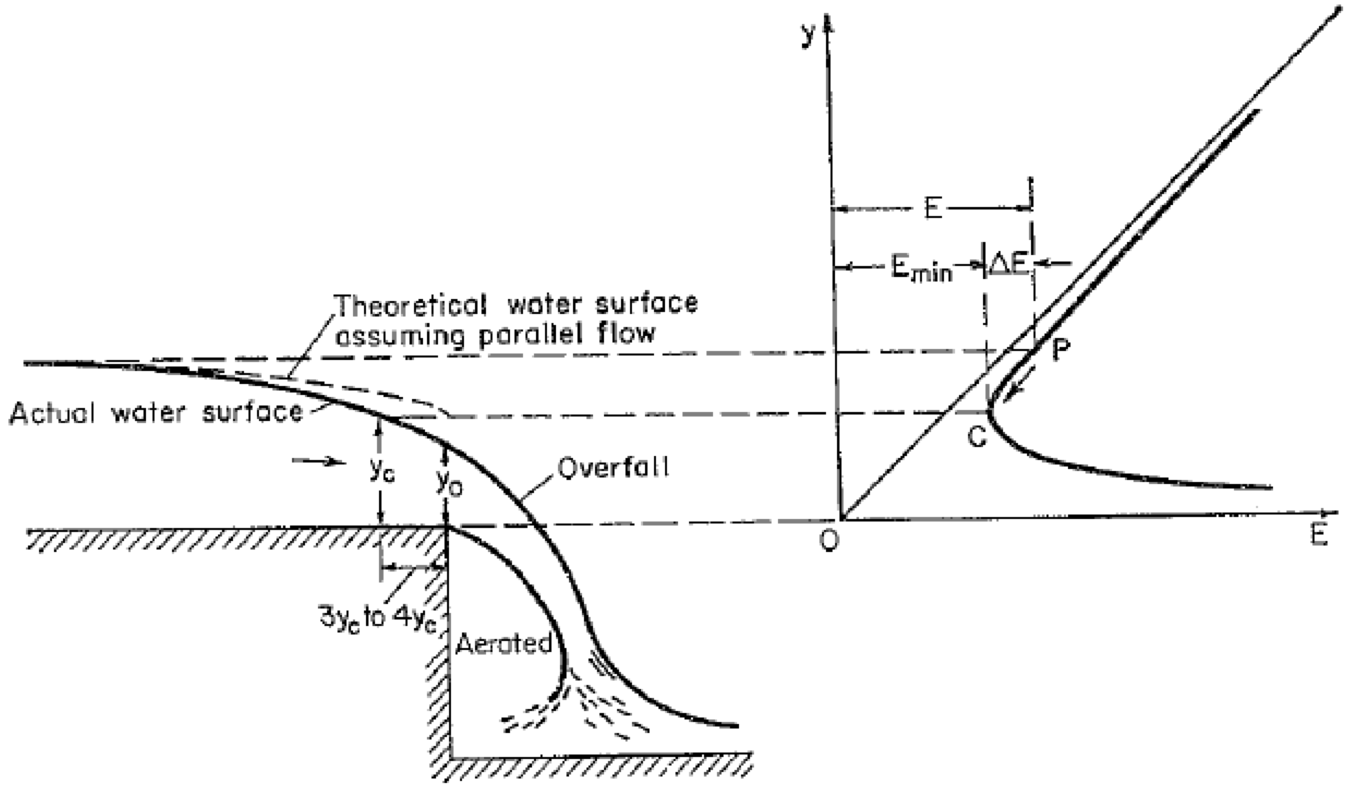
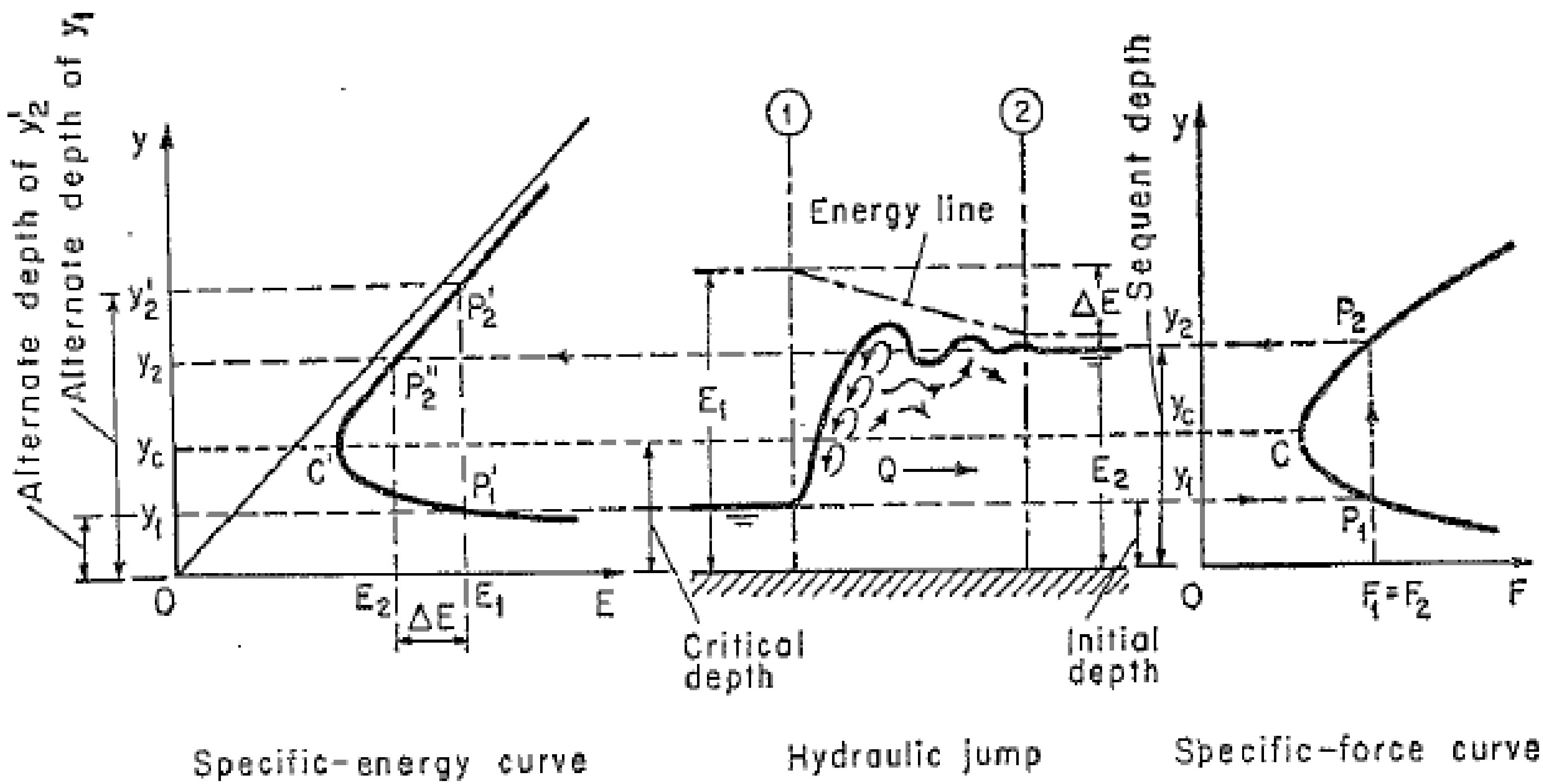
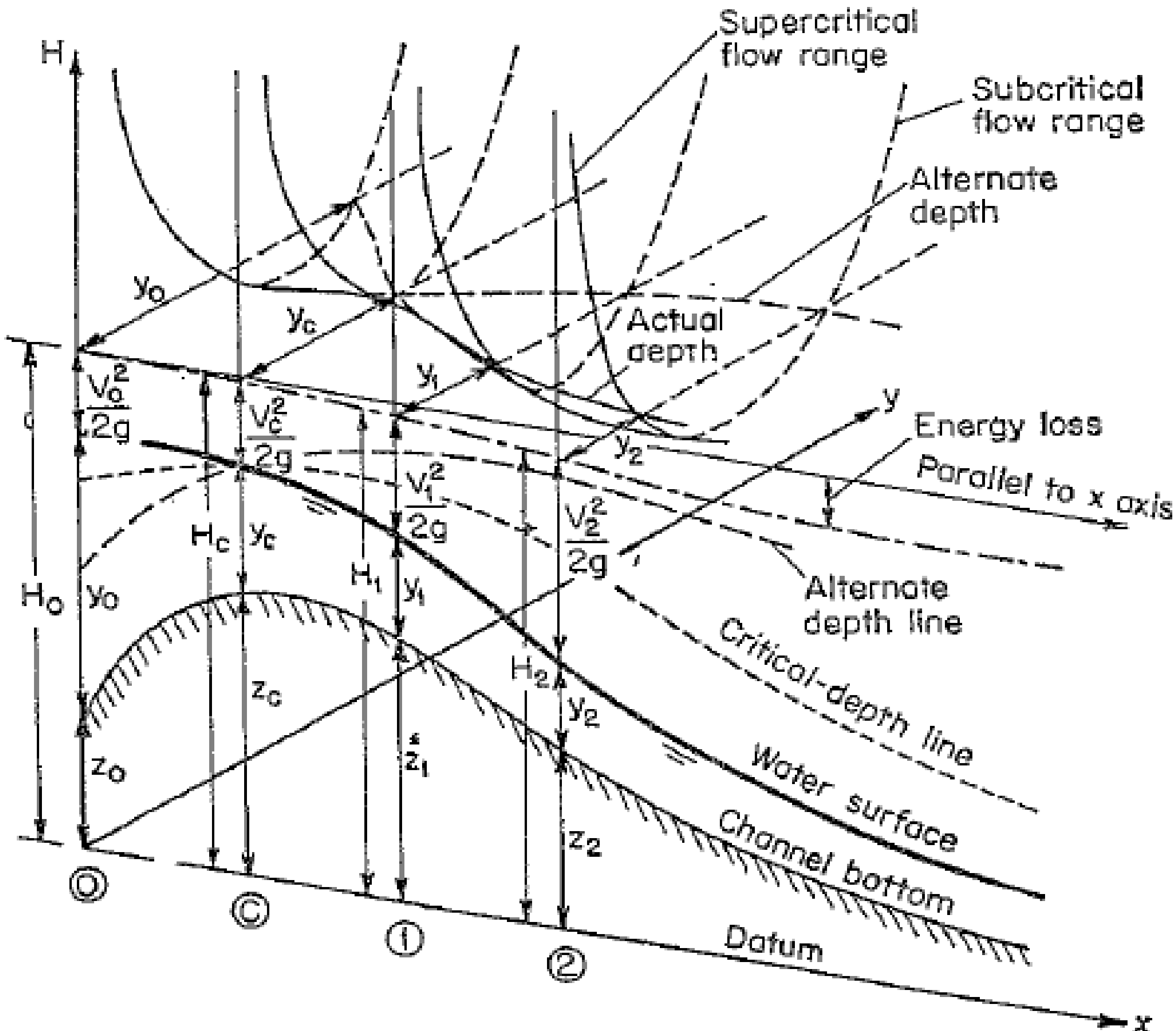
Interpretation of local phenomena using Specific energy curve



Hydraulic Jump



Variation of specific energy-depth in non prismatic channel section



CONTENTS

canal

Transition

Q →

1.0 Channel Transition

1.1 Channel with Hump

Subcritical and Supercritical flow

2.1 Channel with width contraction

Subcritical and supercritical flow

3 Numerical problem discussion

4 Condition for critical flow in control section

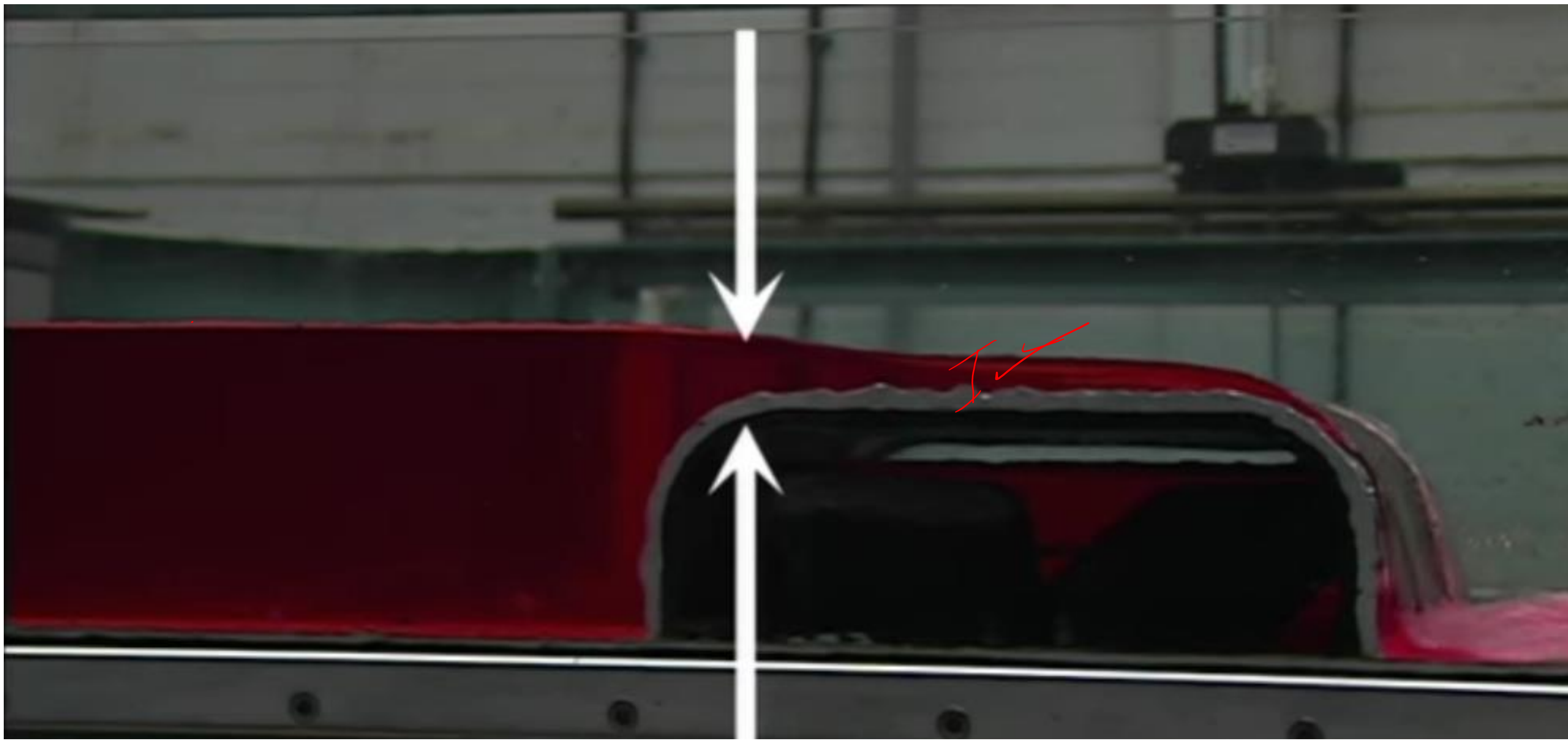
Contd:

Whenever two different x-section of channels are joined to each other without appreciable loss of head, there is a need of an intermediate section which increases/decreases gradually and connects to each other section of channel. This intermediate section of channel is known as transition. Contraction or expansion in width of channel and rise in bed level are type of transition.

Contd:

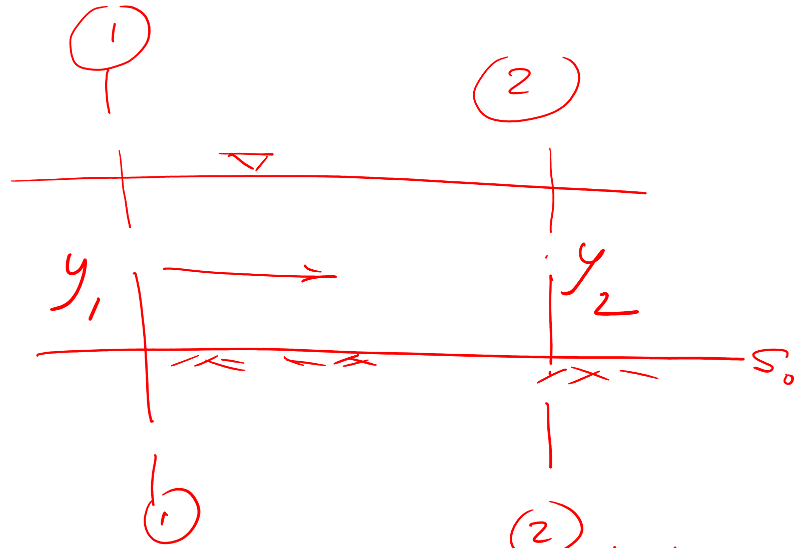
The concepts of specific energy and critical depths are extremely useful in the analysis of problems connected with transitions.

Channel with hump (raised bed level) in subcritical flow condition



Let us consider a channel of fix width B , flowing with discharge Q, with rise in bed level by ΔZ in certain reach as shown in figure below and the flow is subcritical.

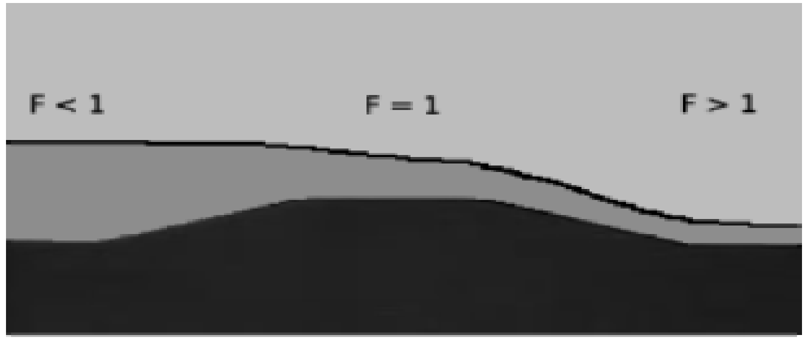
1. Case I (When $\Delta Z < \Delta Z_c$)
2. Case II ($\Delta Z = \Delta Z_c$)
3. Case III ($\Delta Z > \Delta Z_c$)



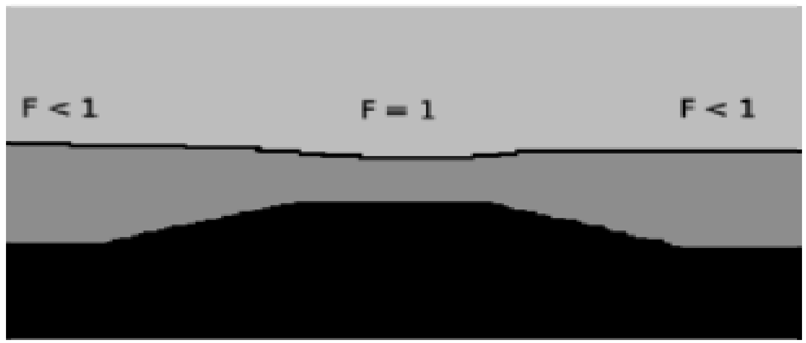
no raised bed

Consider no any losses ~~in flow~~

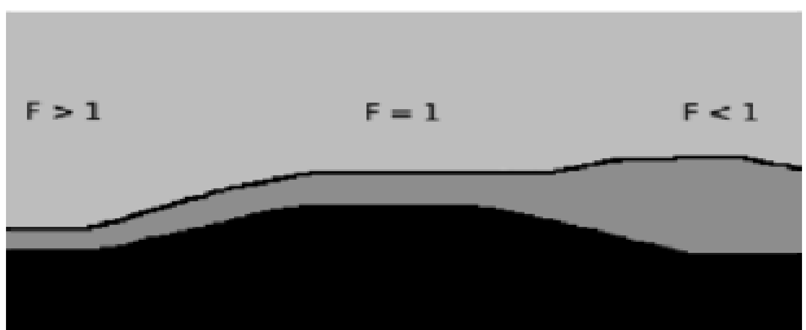
i.e. $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$



(a)

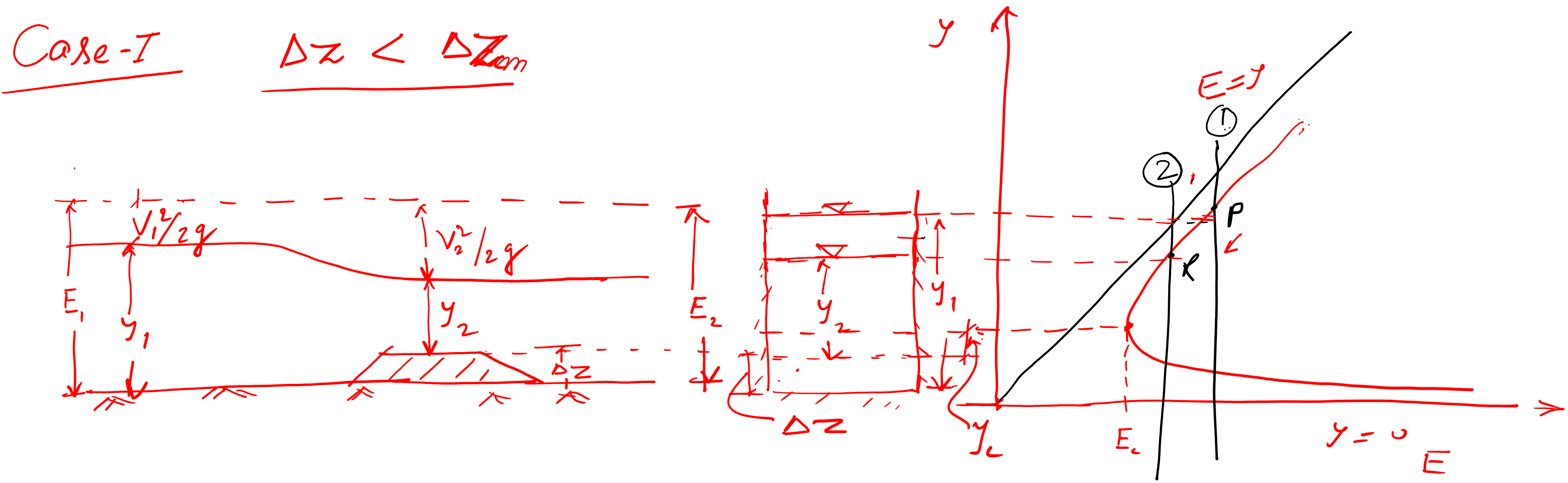


(b)



(c)

Case-I $\Delta z < \Delta Z_m$



assuming no loss

we have,

$$\left(y_1 + \frac{V_1^2}{2g}\right) = \left(y_2 + \frac{V_2^2}{2g}\right) + \Delta z$$

$$E_1 = E_2 + \Delta z$$

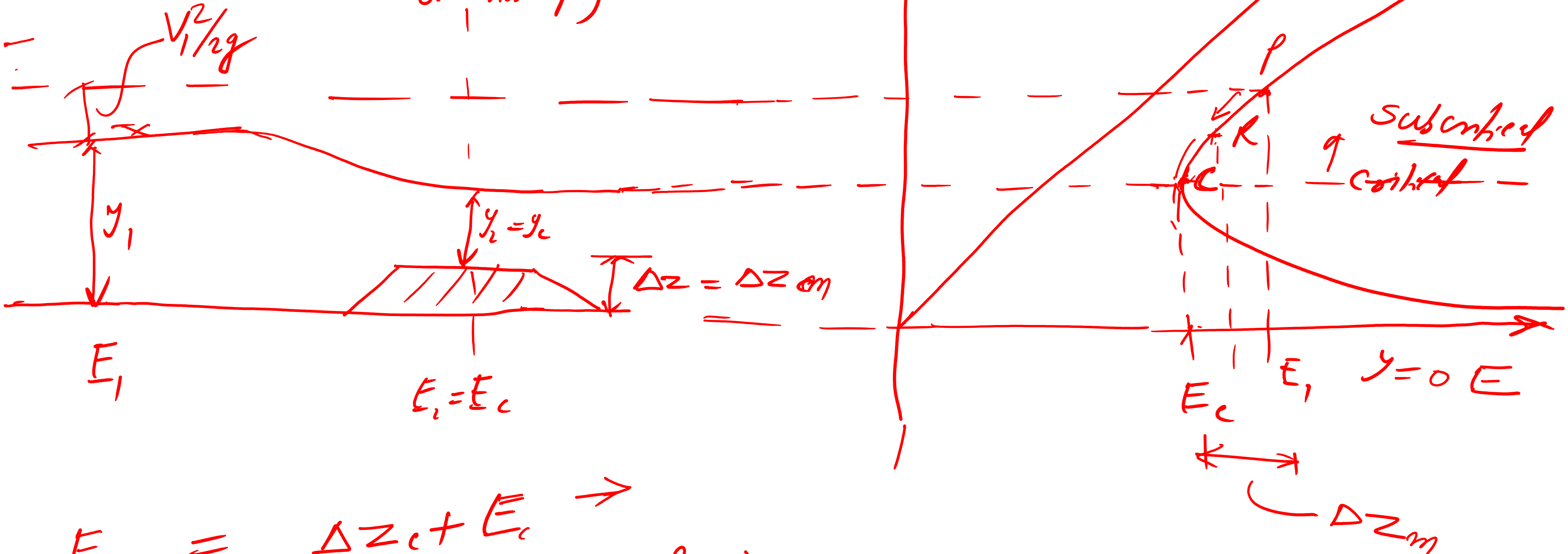
$$\boxed{E_1 - \Delta z = E_2}$$

Let us consider a channel section, Q be discharge and ΔZ be the hump height with which bed level is raised at section (2).

case II

farther increase hump height equals to ΔZ_m ($\Delta Z = \Delta Z_m$)

(Minimum height of hump for critical depth occurrence over hump)



$$E_1 = \Delta Z_c + E_c$$

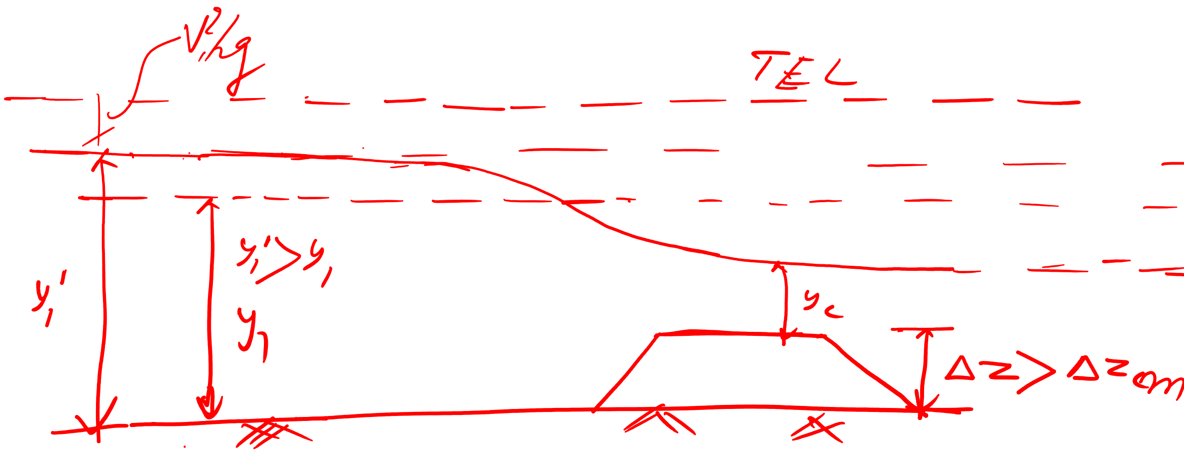
$$E_1 = \Delta Z_c + \left(y_c + \frac{V_c^2}{2g}\right) = \Delta Z_c + \frac{3}{2} y_c$$

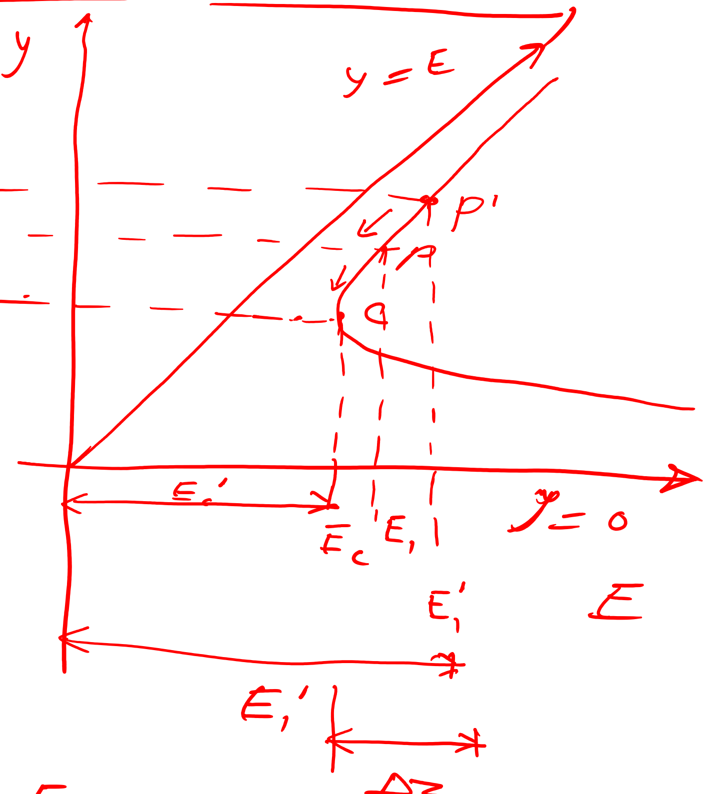
a) $$y_1 + \frac{q^2}{2 g y_1^2} = \Delta Z_c + \frac{3}{2} y_c$$

$$q = Q/B$$

Case – III

Let us further increase hump height such that $\Delta z > \Delta z_m$





E_1

$E_1' > E_1$

$$E_1' = y_1' + \frac{V_1'^2}{2g}, \quad E_2 = \Delta z + E_c$$

applying energy of section (1) & (2)

$E_1' = E_2$ or

$$y_1' + \frac{V_1'^2}{2g} = \Delta z + y_c + \frac{V_c^2}{2g}$$

$$\left(y_1' + \frac{q^2}{2g y_1'^2}\right) = \Delta z + \frac{3}{2} y_c \qquad \text{—— (iii)}$$

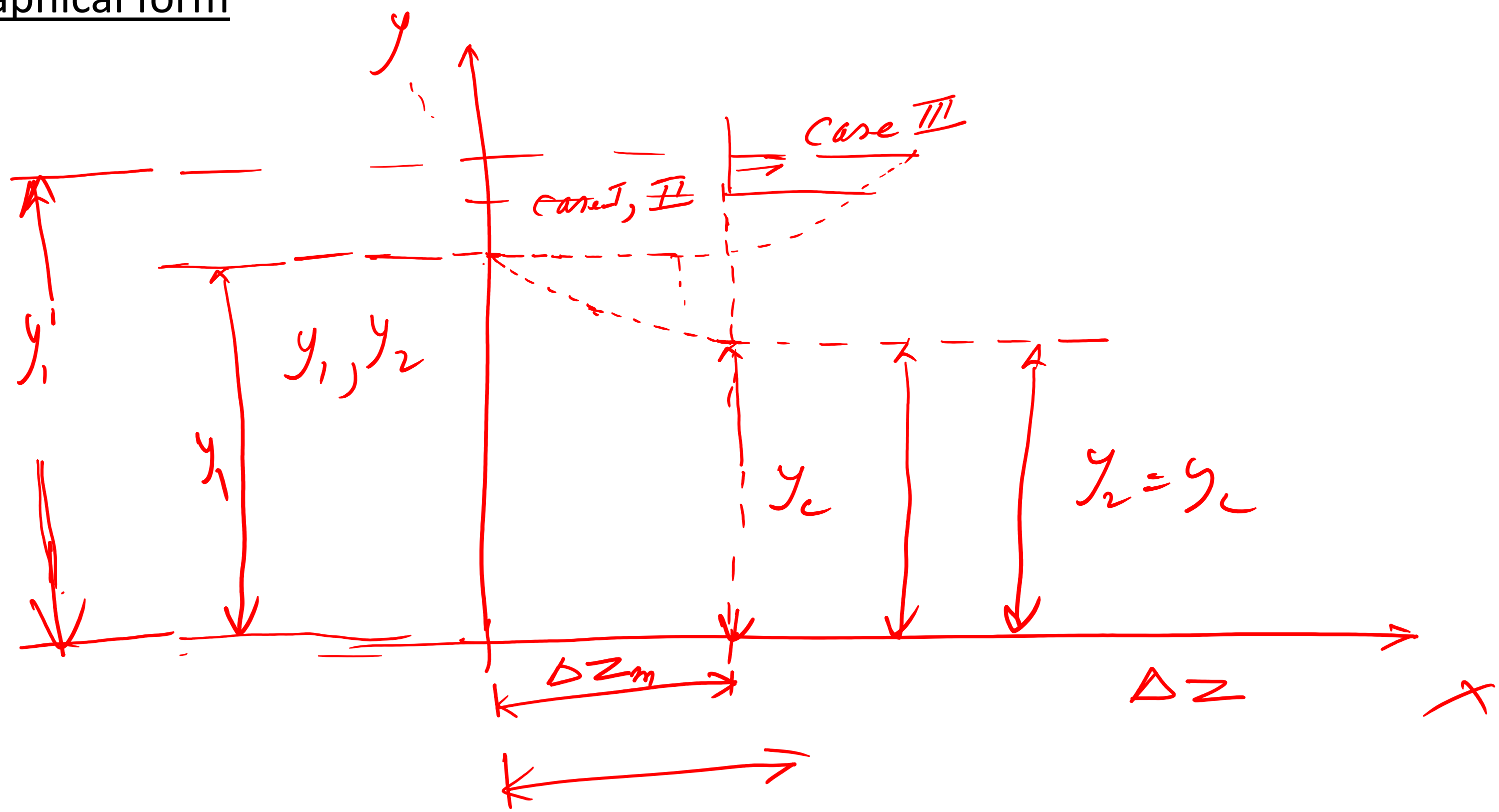
$$y_1' = \square \ ?$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}, \quad q = Q/B$$

$$\Delta z = \text{given}$$

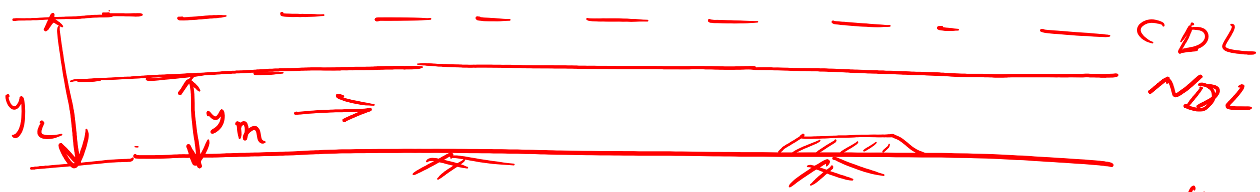
https://youtu.be/akzE0k9Dqxw

Summary of all three cases for subcritical flow condition in graphical form



A hump in Supercritical flow

The flow parameter in supercritical flow at hump will be similar as subcritical flow but the changes are of opposite sign.i.e., The effect of hump in supercritical flow will be opposite to Let's discuss ,



$y_n < y_c$

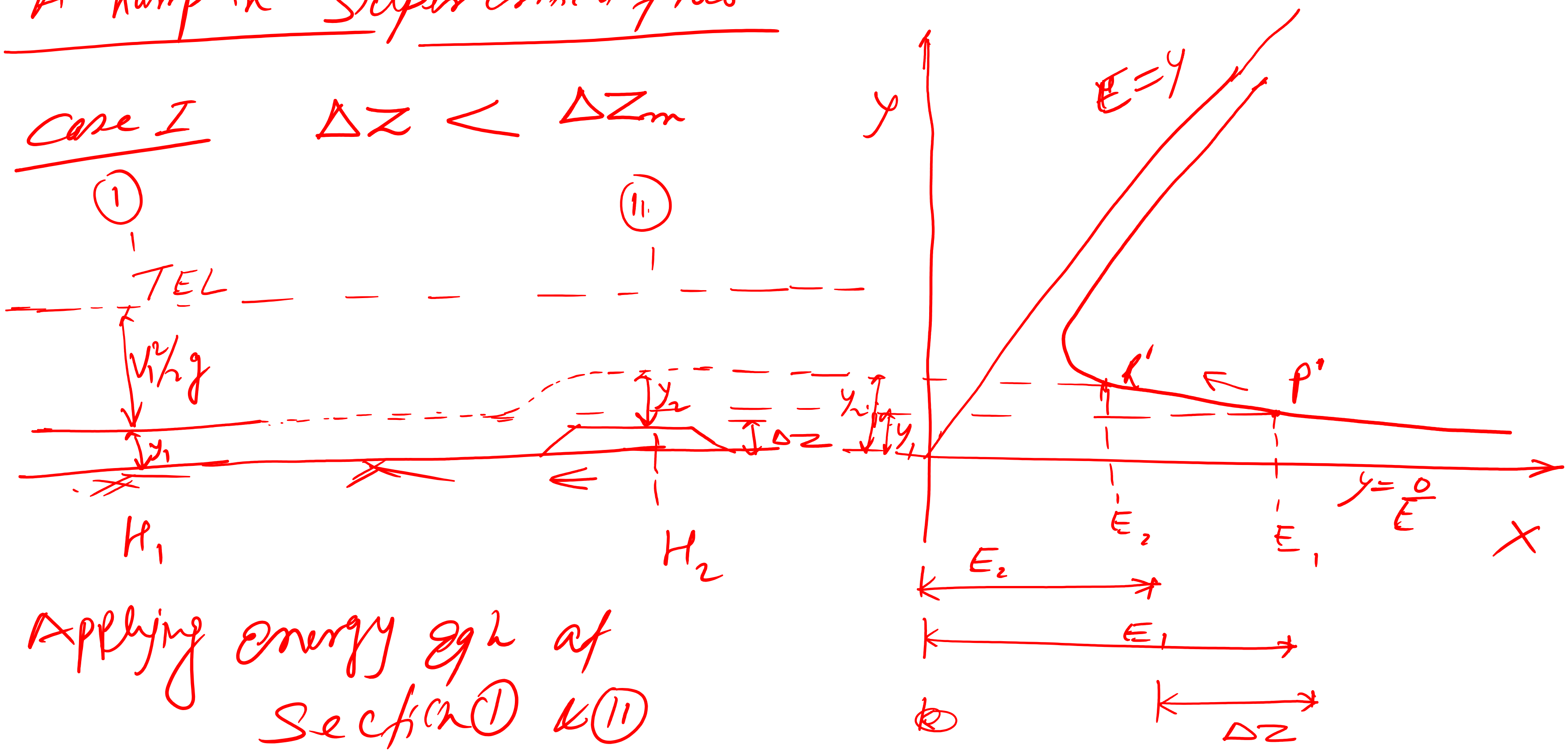
$Fr > 1$

Supercritical flow

Case I (when $\Delta Z < \Delta Zm$)

A hump in supercritical flow

Case I $\Delta Z < \Delta Z_m$



Applying energy eqn at Section 1 & 11

$$H_1 = H_2$$

$$\Rightarrow E_1 = \Delta Z + E_2$$

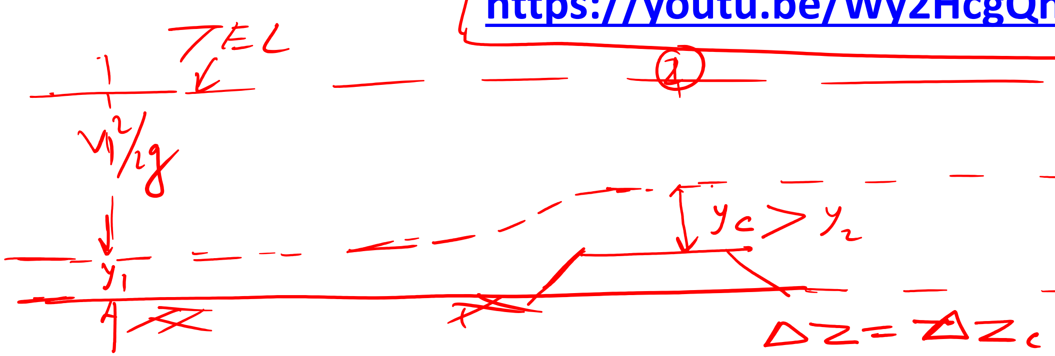
$$y_1 + V_1^2/2g = \Delta Z + y_2 + V_2^2/2g$$

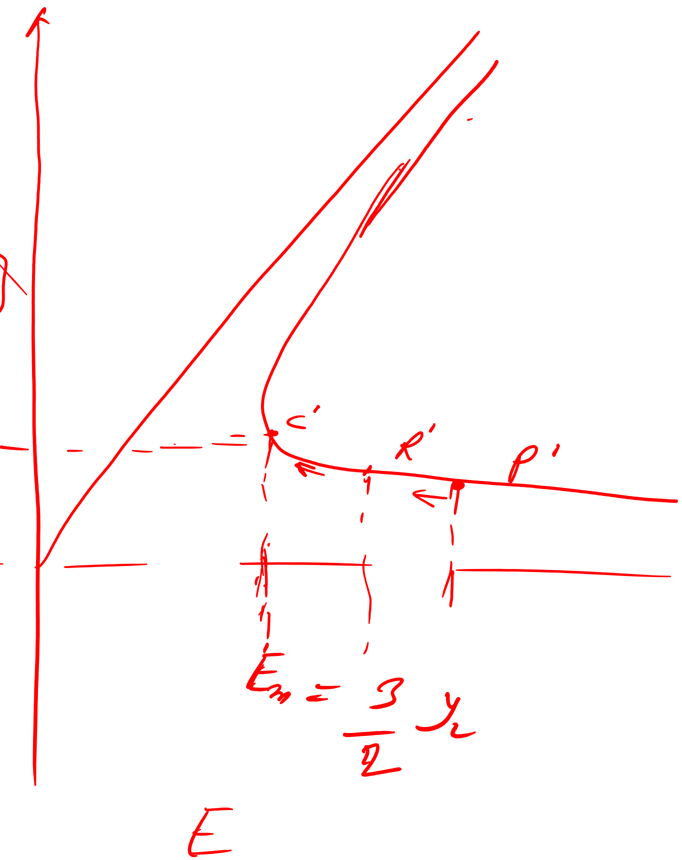
$$y_1 + \frac{q^2}{2gy_1^2} = \Delta Z + y_2 + \frac{q^2}{2gy_2^2} \quad) \; q = Q/B$$

Case-II) When $\Delta Z = \Delta Z_m$

①

https://youtu.be/Wy2HcgQntTM





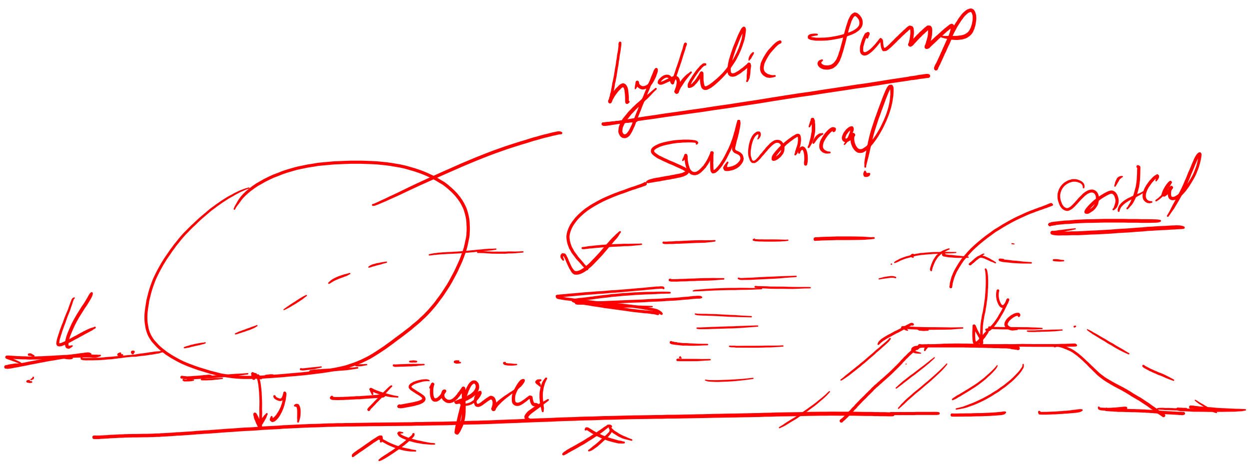
$$y_1 + \frac{q^2}{2gy_1^2} = \Delta Z_m + y_c + \frac{q^2}{2gy_c^2}$$

$$y_1 + \frac{q^2}{2gy_1} = \Delta Z_m + \frac{3}{2} y_c$$

Case III

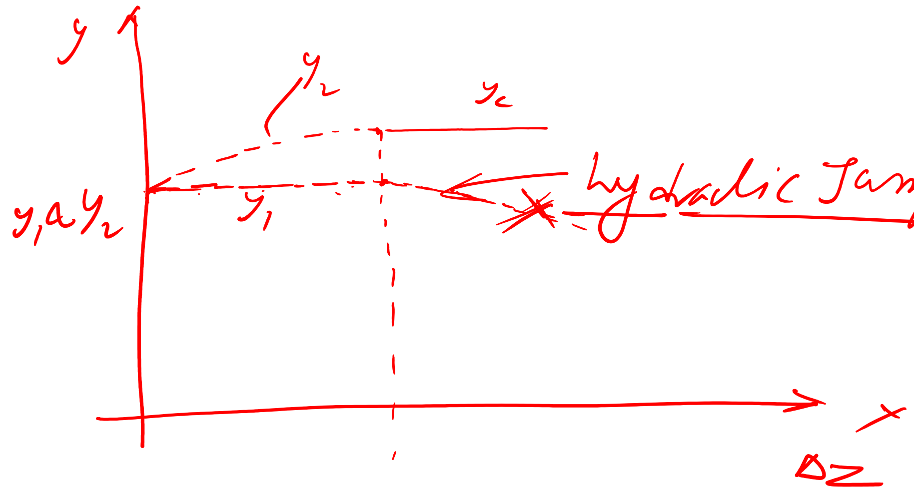
Let us further increase height of hump above minim height, ΔZ_m

$$\Delta Z > \Delta Z_m$$



We can prove that the flow depth at hump will be critical always

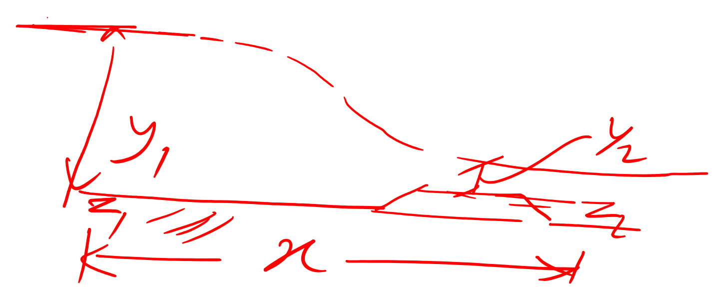
Summary of all three cases



Total energy

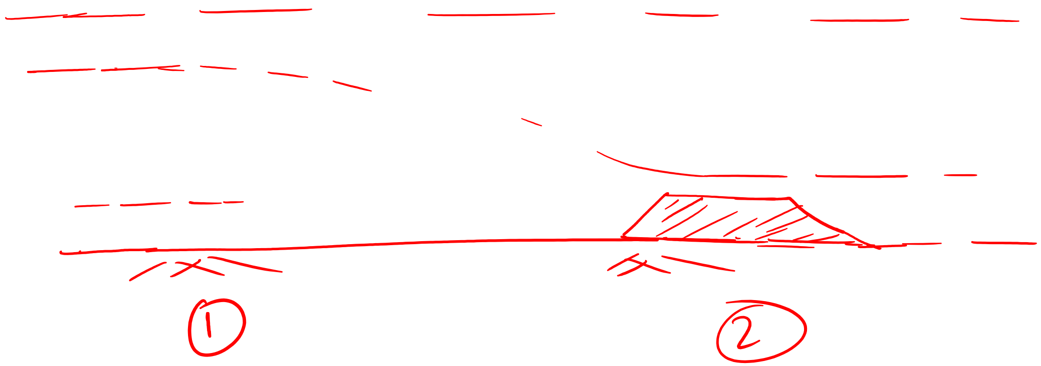
$$H = z + y + \frac{V^2}{2g}$$

(ii) $$H = z + y + \frac{Q^2}{2gA^2}$$



$$\frac{(y_1 - y_2)}{(x_2 - x_1)} = \frac{dy}{dx}$$

$$\left(\frac{z_2 - z_1}{x_2 - x_1}\right) = \frac{dz}{dx}$$



$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g}\,\frac{d\left(\frac{1}{A^2}\right)}{dx}$$

$$= \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g}\left(\frac{dA^{-2}}{dx}\right)$$

$$= \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g}\left(\frac{dA^{-2}}{dy} \times \frac{dy}{dx}\right)$$

$$= \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g}\left(-2A^{-3} \times \frac{dA}{dy}\,\frac{dy}{dx}\right)$$

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} - \frac{Q^2 T}{g A^3} \frac{dy}{dx}$$

F_r^2

$$\frac{dA}{dy} = T$$

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx}\left[1 - F^2\right]$$

In ~~case~~ our case TEL = const y = const

thus $\frac{dH}{dx} = 0$

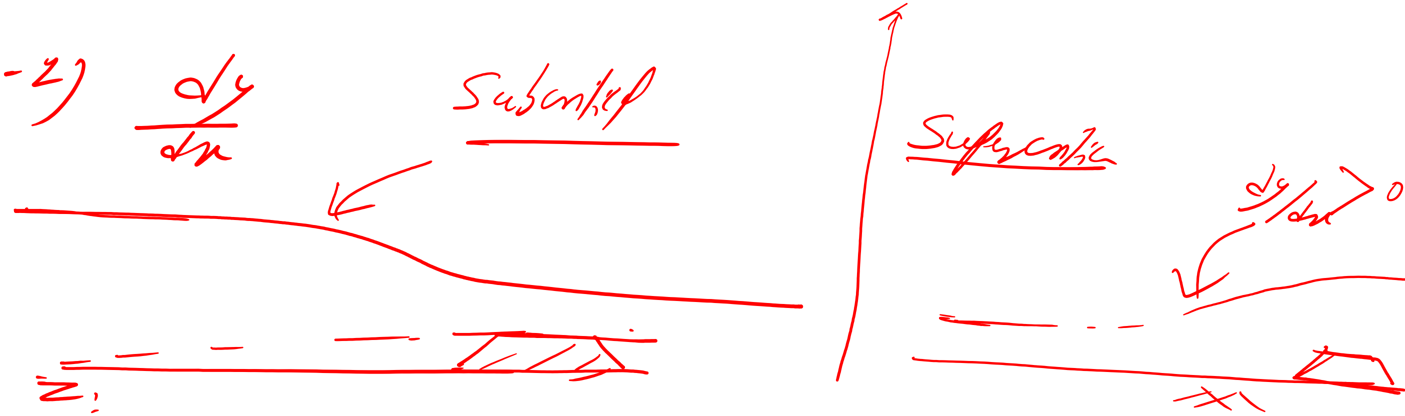
$$0 = \frac{dZ}{dx} + \frac{dy}{dx}(1 - F^2)$$

$$\frac{dZ}{dx} = (F^2 - 1)\frac{dy}{dx}$$

subcritical flow

$$\frac{dz}{dx} = (F^2-1)\frac{dy}{dx}$$

Subcritical flow



$$\frac{(Z_2 - Z_1)}{x_2 - x_1} = \frac{dz}{dx} > 0$$

$$\frac{dZ}{dx} > 0$$

So $(F^2-1)\frac{dy}{dx} > 0$

then

both (F^2-1) & $\frac{dy}{dx} > 0$ (both positive)

$\quad$ — $\quad$ — < 0 (both -ve)

$(F^2-1) > 0 \qquad dy/dx > 0$

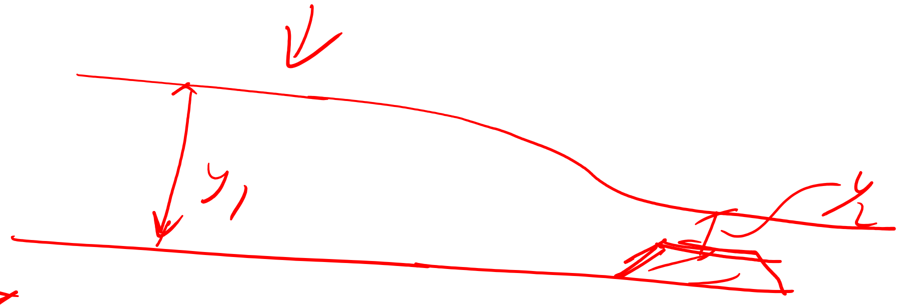
$F > 1$, $dy/dx > 0 \Rightarrow$ supercritical case

Second case

$(F^2-1) < 0$ & $\frac{dy}{dx} < 0$ $\qquad \frac{y_2-y_1}{x_2-x_1} = -ve$

$F^2-1<0$

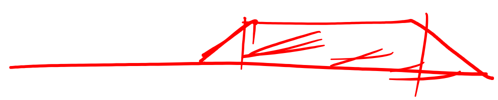
$F<1$



$$\frac{dZ}{dx} = (F^2-1)\frac{dy}{dx} \qquad \frac{dy}{dx} < 0$$

at hump

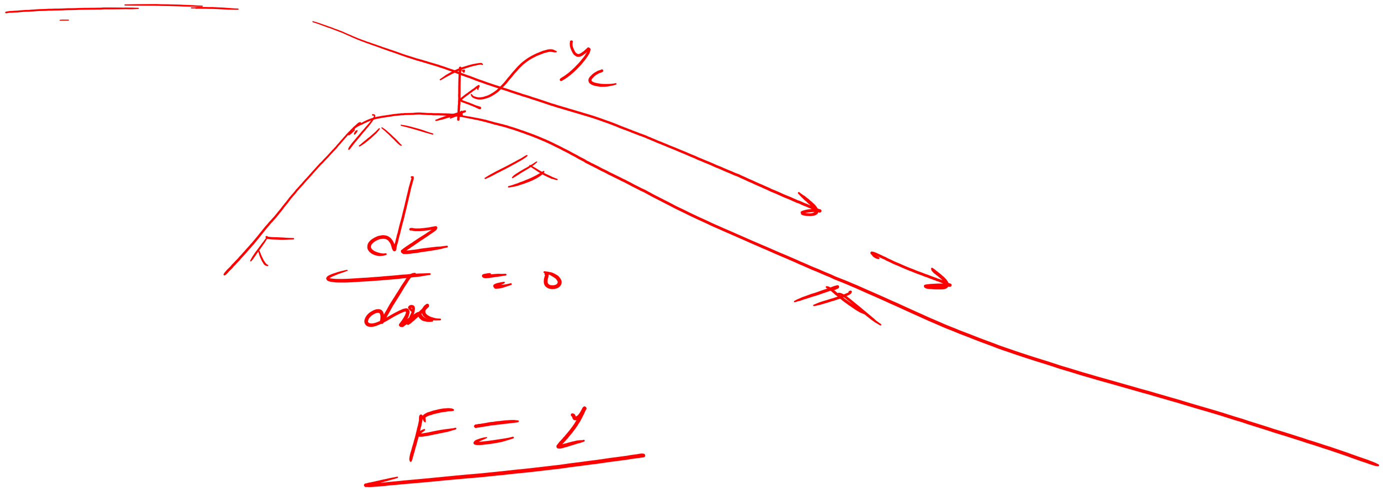
$\frac{dZ}{dx} = 0$ $\qquad dy/dx = 0$



$(F^2-1)\frac{dy}{dx} = 0$ $\qquad dZ/dx = 0$

$(F^2-1) = 0$

$F^2 = 1 \Rightarrow F = 1$ = flow is critical



$$\frac{dz}{dx} = 0$$

$$F = 1$$

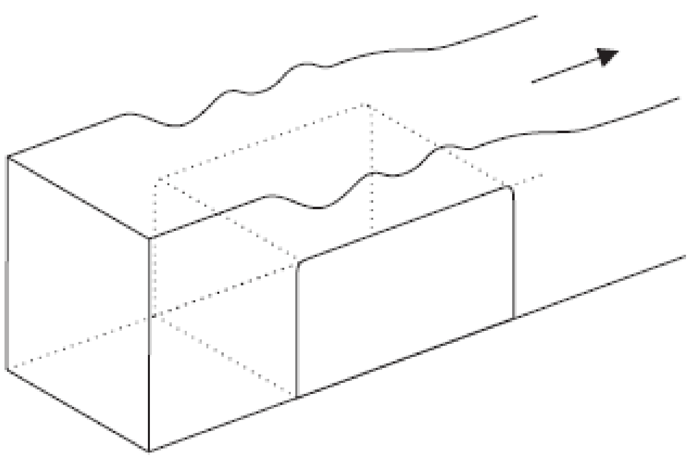
Numerical

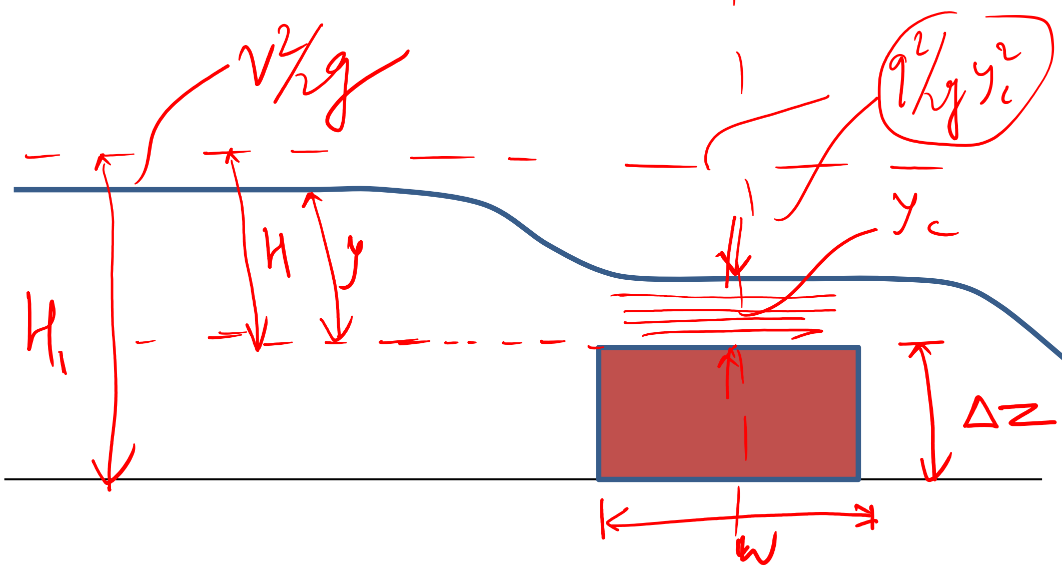
Application of concept of channel transition with hump

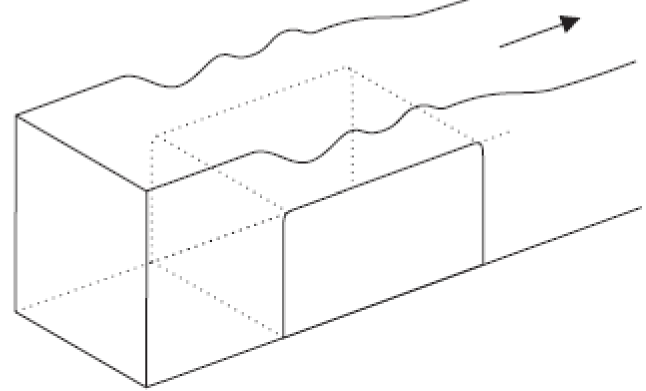
Broad crested weir

If the channel floor is further raised up to height equals to or greater than the minimum height of hump ΔZm over a length sufficient enough for parallel flow to occur over the hump, as already discussed, the flow over hump will be critical. Such a structure is called as broad crested weir.

A broad-crested weir is a flat-crested structure with a crest length large compared to the flow thickness. The ratio of crest length to upstream head over crest must be typically greater than 1.5-3(e.g. Chow, 1973; Henderson, 1966)







$$\Delta z > \Delta z_m$$

$$y_c + \frac{q^2}{2g y_c^2} = y_1 + \frac{V^2}{2g} = H$$

$$H = \Delta z + y + \frac{V^2}{2g}$$

$$\frac{W}{H - \Delta z} > 1.5 - 3$$

at hump,

$y_c + q^2/2gy_c^2 = y_1 + \frac{V_1^2}{2g} = H$

$$y_c = \left(q^2/g\right)^{1/3}$$

$$\left(E = \frac{3}{2}y_c\right)$$

$$H = y_c + \frac{q^2}{2g y_c^2}$$

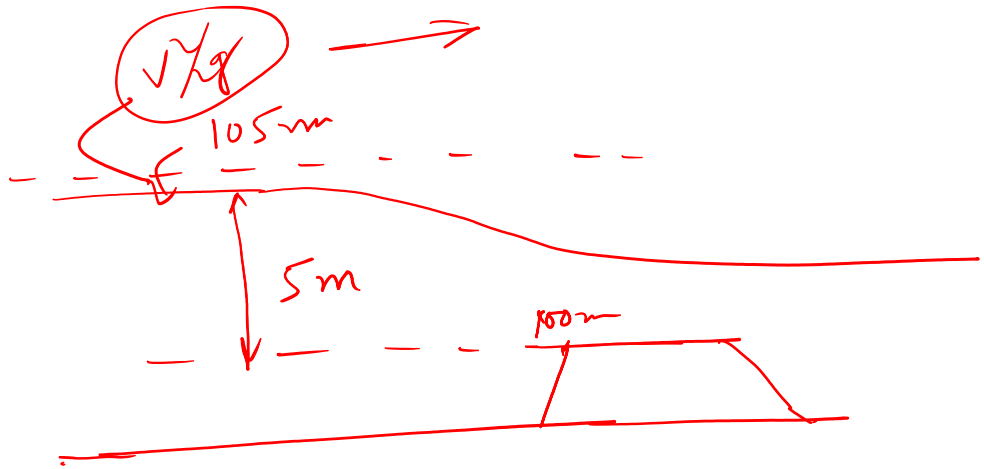
$$H = \frac{3}{2}y_c = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$$

or $$H = \frac{3}{2}\left(\frac{\left(\frac{Q}{B}\right)^2}{g}\right)^{1/3}$$

$$Q = \left(\frac{2}{3}\right)^{3/2} B\sqrt{g}\,H^{3/2}$$

$$\boxed{Q = 0.544\,B\sqrt{g}\,H^{3/2}}$$

Discharge formula of broad crested weir



5.0012

$$Q = 0.544\, B\sqrt{g}\, H^{3/2}$$

$$= 0.544\, B\sqrt{g}\,(5)^{3/2}$$

Numerical

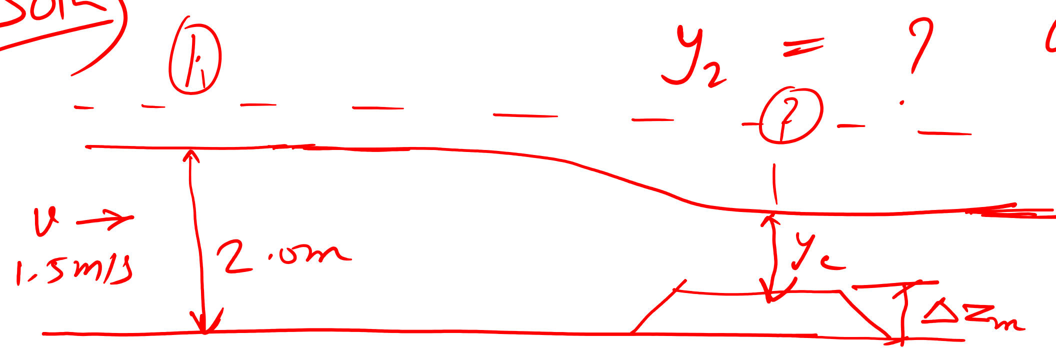
Q 1)

Water flow at a depth of 2.0m and a velocity of 1.5m/s in a 4.0m wide channel. Find the height of hump required to produce critical flow without affecting upstream depth. (ii) The depth over the hump when the height of hump is half the above value.

$\Delta Z_m = ?$

$y_2 = ?$ when $\Delta Z = \frac{\Delta Z_m}{2}$

Soln)



$$\frac{V_2^2}{2g} = \frac{Q^2}{2gB^2y^2} = \frac{q^2}{2gy_c^2}$$

applying energy eqn at section (1) & (2)

$$y_1 + \frac{V_1^2}{2g} = \Delta Z_m + y_c + \frac{V_2^2}{2g}$$

$$= \Delta Z_m + y_c + \frac{q^2}{2gy_c^2}$$

$$y_1 + \frac{V_1^2}{2g} = \Delta Z_m + \left(y_c + \frac{q^2}{2g y_c^2}\right) \qquad q = Q/B$$

$$E_m = \frac{3}{2} y_c$$

$$y_1 + \frac{V_1^2}{2g} = \Delta Z_m + \frac{3}{2} y_c \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(q^2/g\right)^{1/3}$$

but $q = Q/B = \dfrac{A_1 V_1}{B}$

$$= \frac{4 \times 2 \times 1.5}{4} = 3\ m^3/s/m$$

$$\boxed{y_c = 0.97}$$

$$y_c < y_1\ (2m)$$

finally

$$\Delta Z_m = 0.656\ m$$

$$\Delta z = \frac{\Delta z_m}{2} = \frac{0.656}{2} = \underline{\underline{0.328}}$$

$$y_1 + \frac{v_1^2}{2g} = \Delta z + y_2 + \frac{q^2}{2g y_2^2}$$

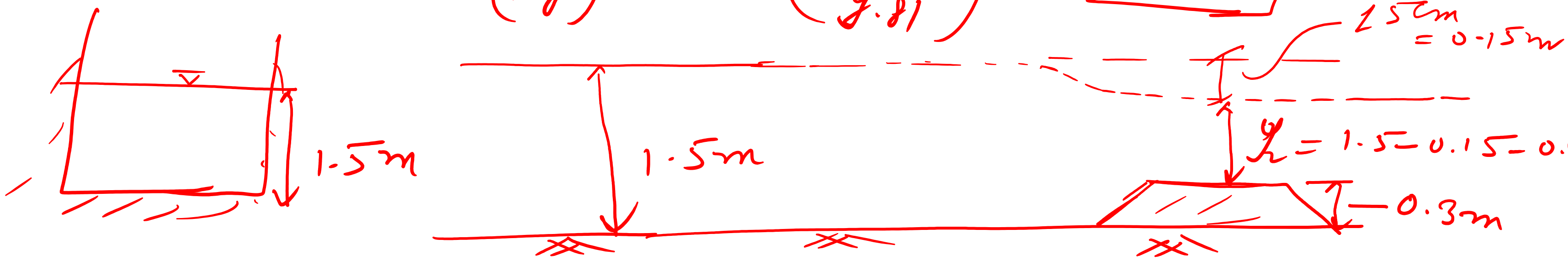
$$2 + \frac{(1.5)^2}{2\times 9.81} = 0.328 + y_2 + \frac{(3)^2}{2\times 9.81 \times y_2^2}$$

solving above

we get $y_2 = 1.62\,m$

Q2) Water flows in a rectangular channel at a depth of 1.5m. A 30 cm high smooth hump produces a drop of 25cm in the water surface elevation. Neglect losses. estimate the discharge per unit width of the channel. (9)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(2.522)^2}{9.81}\right)^{1/3} = \underline{0.86\,m}$$



$$\frac{Q}{B} = q = ? \quad (\text{Discharge Per unit width of channel})$$

$$y_2 = 1.5 - 0.15 - 0.3$$
$$= 1.5 - 0.45$$
$$= \underline{1.05\,m} \quad y_2 > y_c$$

$$y_1 + \frac{v^2}{2g} = \Delta z + y_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{By}$$

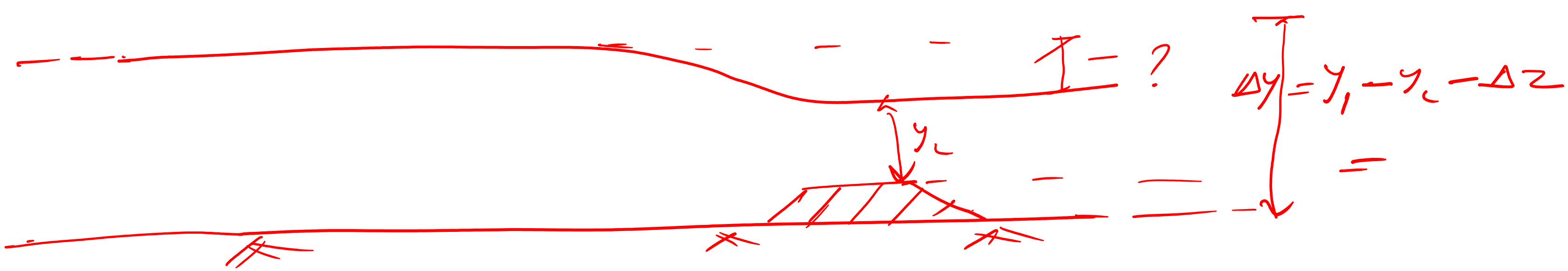
$$q_1 = q_2$$

$$y_1 + \frac{q^2}{2g y_1^2} = \Delta z + y_2 + \frac{q^2}{2g y_2^2}$$

$$q = \square$$

$$= \underline{2.522\ m^3/s/m}$$

Q3 A 2.25 m wide rectangular channel has a flow with a velocity of 1.35 m/s and a depth of 1.2 m. A smooth hump is to built at a section to create critical flow condition over the hump. calculate (a) the minimum height of hump reqd to achieve this (b) the resulting change in the water surface elevation.



$\Delta z = 0.327$ ft

$\Delta y = y_1 - y_c - \Delta z = 0.22$ m

Today we will discuss on

A Channel contraction

Example 1) Bridge, culvert
2) Barrage
3) venturi flume (To measure discharge)

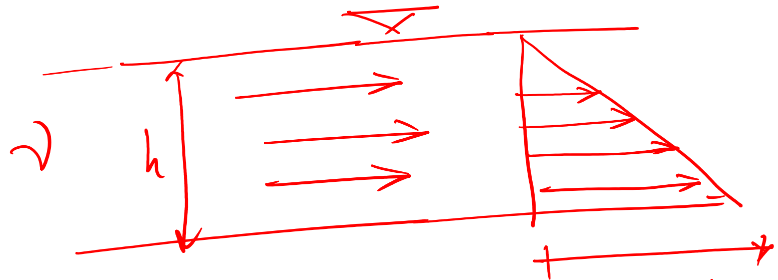
B pressure distribution in open channel.

c) variation of hydraulic Radius with flow depth.

Pressure distribution

straight channel

noncurvilinear



$$p = \rho g h = \gamma h$$

curvilinear channel

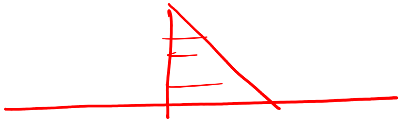
$$a = a_s + a_n$$

$$= \left(v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right) + \frac{v^2}{r}$$

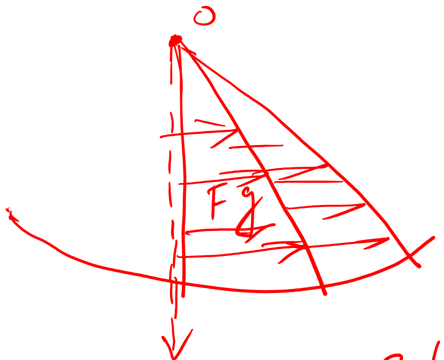
in uniform & steady flow

$$\frac{\partial v}{\partial s} = 0 \quad \& \quad \frac{\partial v}{\partial t} = 0$$

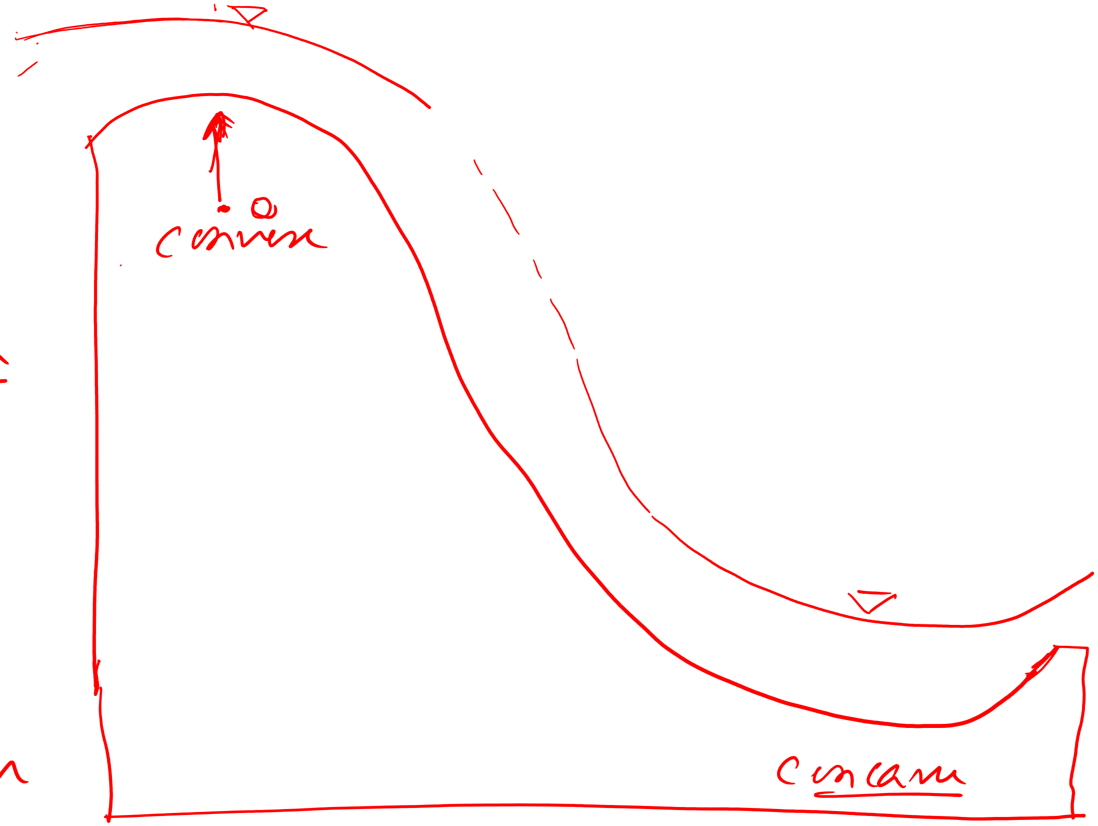
concave surface



$\gamma h = \rho g h$

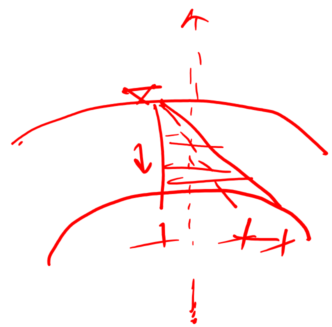


$$p = \rho g h + \rho\left(\frac{v^2}{r}\right)h$$



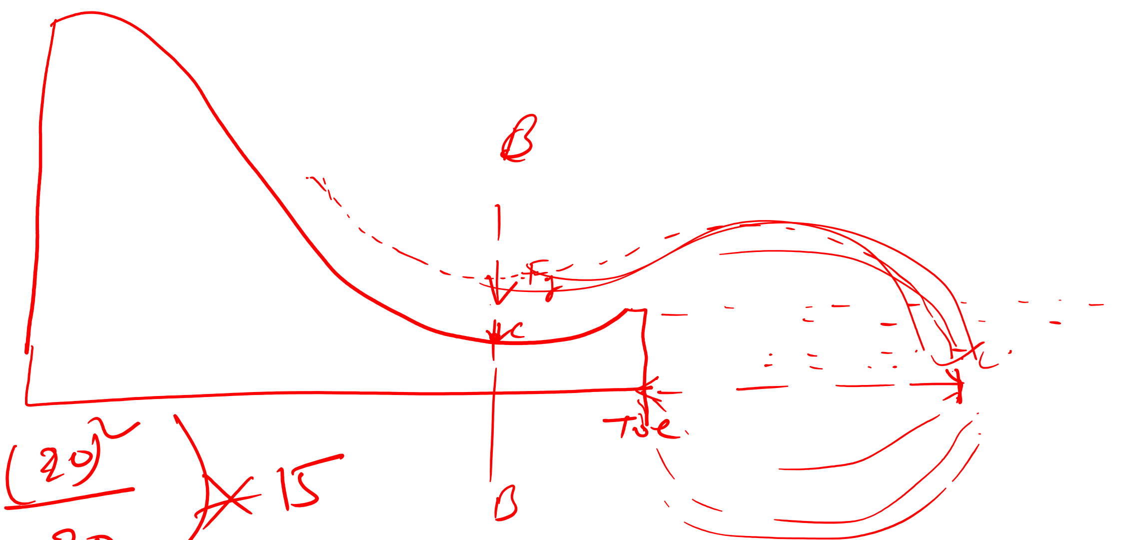
for convex surface

$$p = \rho g h - \rho\left(\frac{v^2}{r}\right)h$$



Q) A spillway flip bucket has a radius of 20 m. If the flow velocity at section B-B is 20 m/s and the flow depth is 15 m. compute the pressure intensity at point C

Soln)



$$p = \rho g h + \rho \left(\frac{v^2}{r}\right) h$$

$$= 1000 \times 9.81 \times 15$$

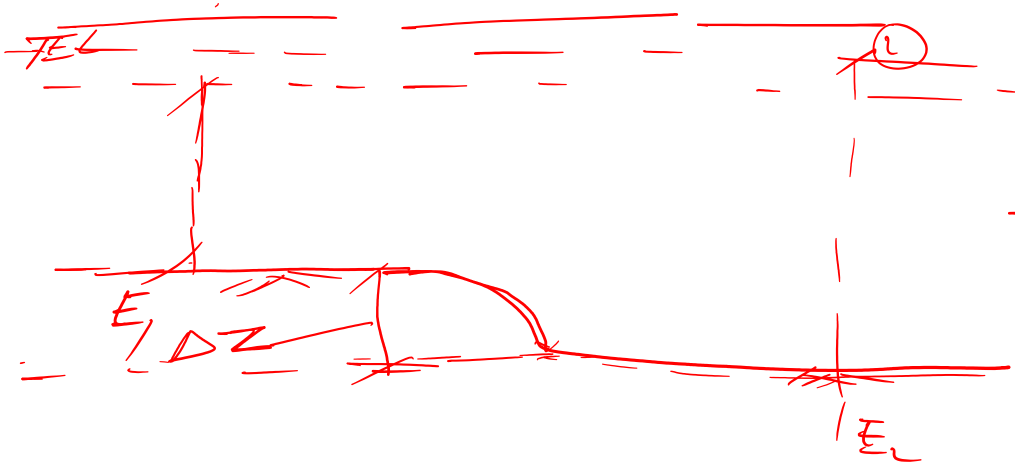
$$+ 1000 \times \left(\frac{(20)^2}{20}\right) \times 15$$

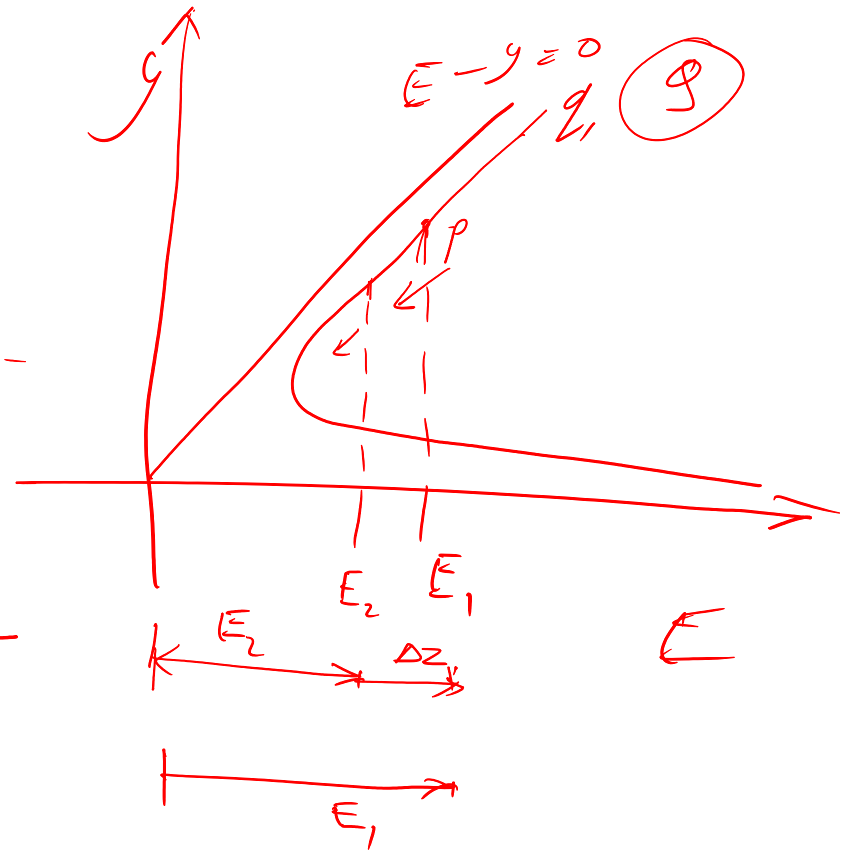
$$= \boxed{\quad} \ N/m^2$$

Channel Transition (contd)

$E_1 = E_2 + \Delta Z$

i.e. $E_2 = E_1 - \Delta Z$





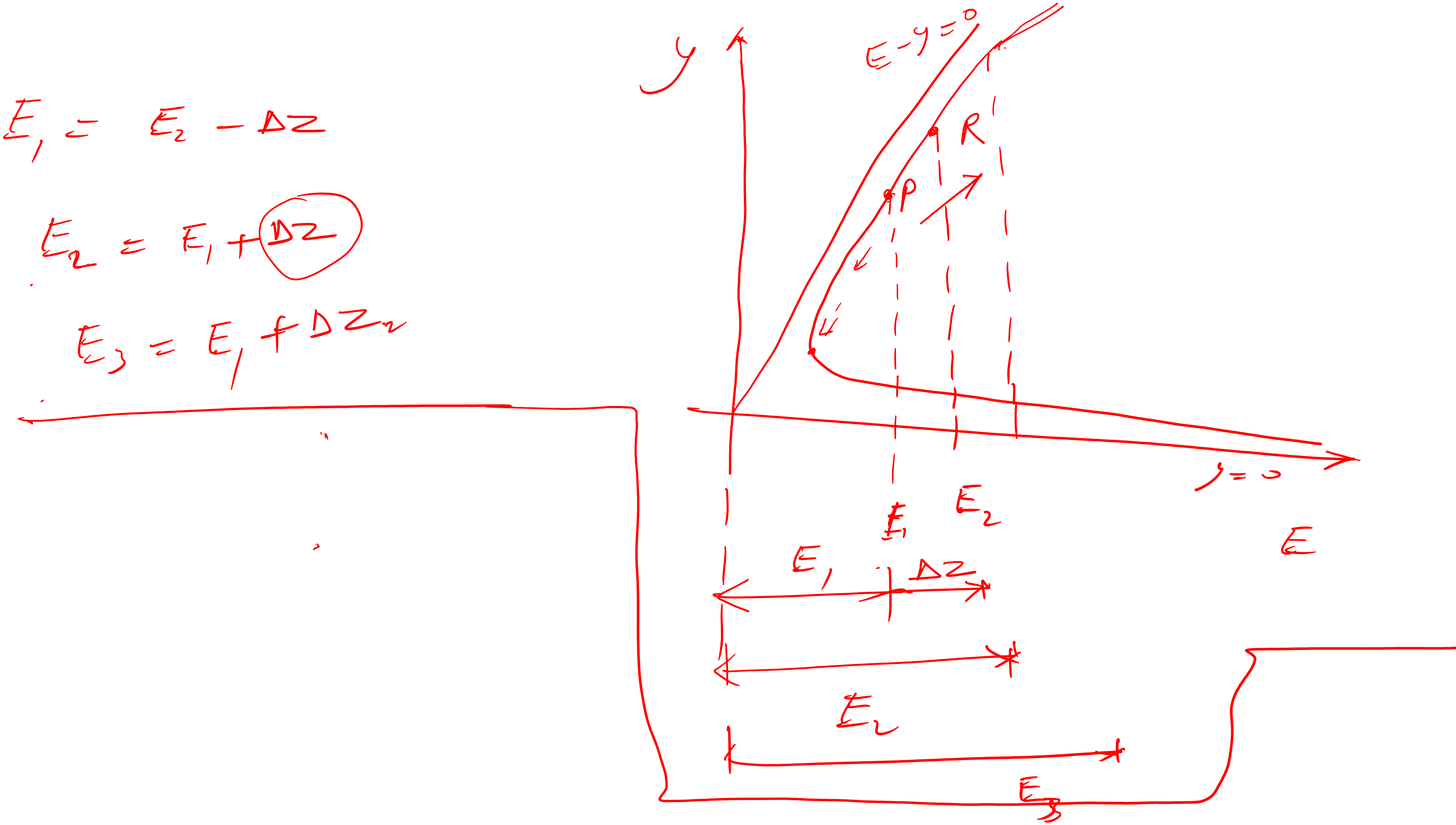
$\Delta Z + E_1 = E_2$

$\Rightarrow E_1 = E_2 - \Delta Z$

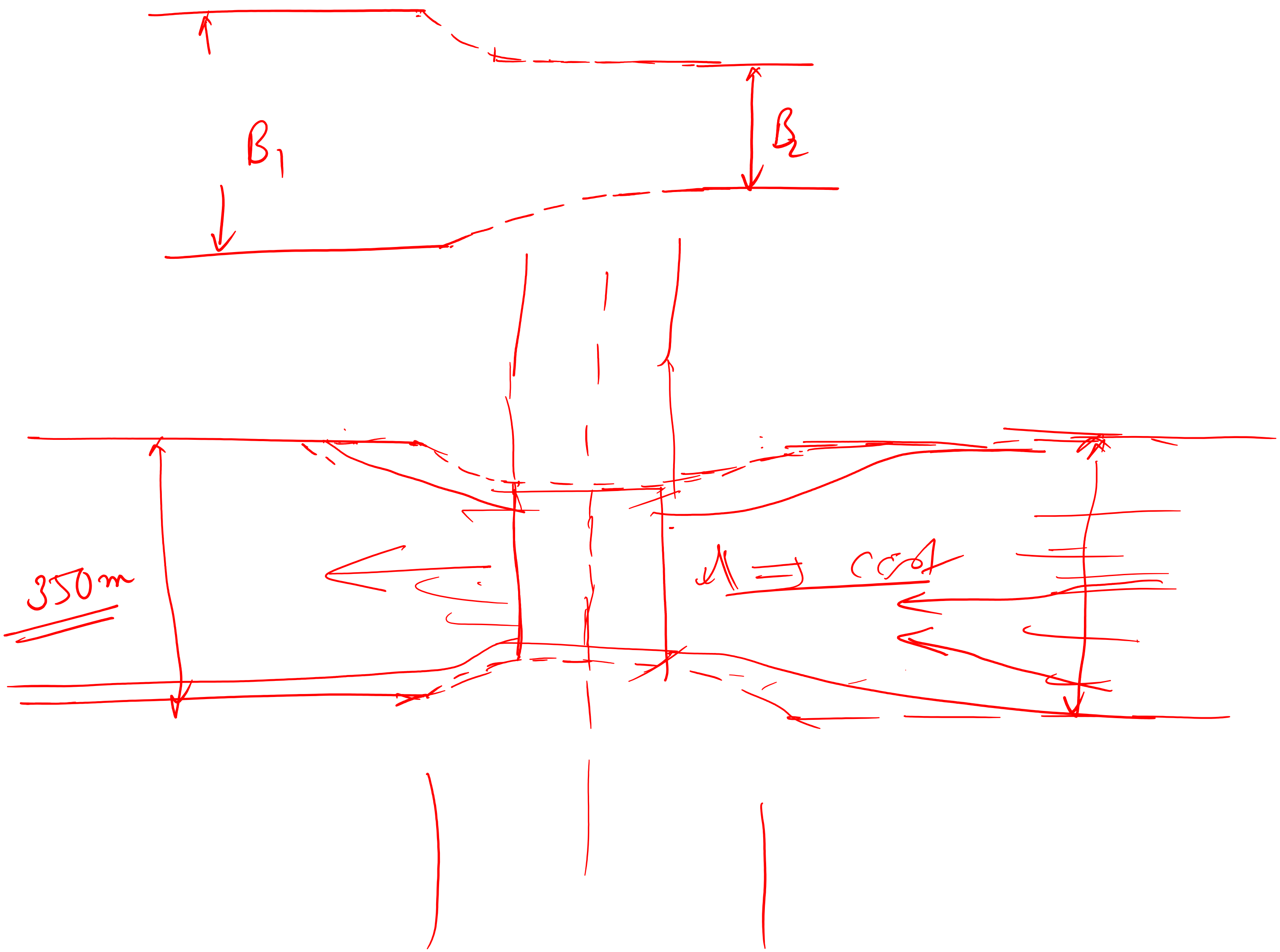
$E_1 = E_2 - \Delta Z$

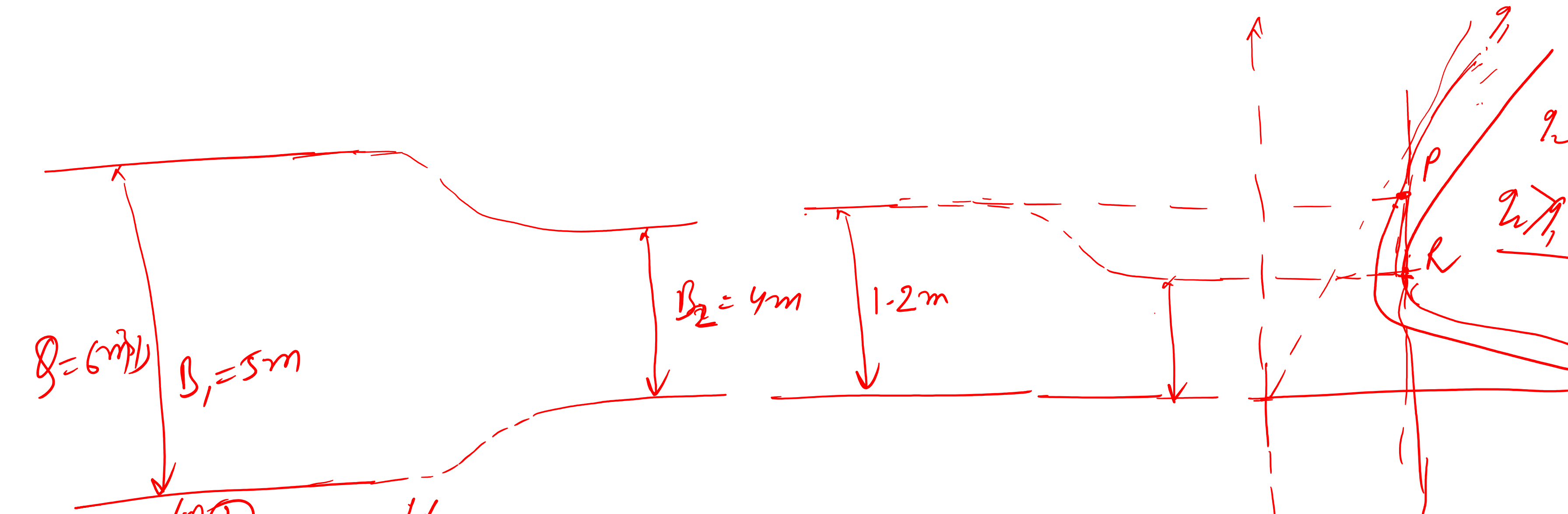
$E_2 = E_1 + \Delta Z$

$E_3 = E_1 + \Delta Z_2$



channel contraction





$E_1 = E_2$

at section ②

$$y_c = \left(\frac{q_2^2}{g}\right)^{1/3} = \underline{\underline{0.612}}$$

$$E_c = \frac{3}{2} y_c = \frac{3}{2} \times 0.612 = \underline{0.918\,m}$$

$$E_1 = y_1 + \frac{q_1^2}{2g y_1^2} = 1.2 + \frac{(1.2)^2}{2 \times 9.81 \times (1.2)^2} = \underline{1.25\,m}$$

$$E_1 > E_c$$

$$E_1 = E_2$$

$$y_1 + \frac{q_1^2}{2g y_1^2} = y_2 + \frac{q_2^2}{2g y_2^2} = \underline{\underline{E_2}} > E_c$$

$$\Rightarrow y_2 = 1.117 > y_c$$

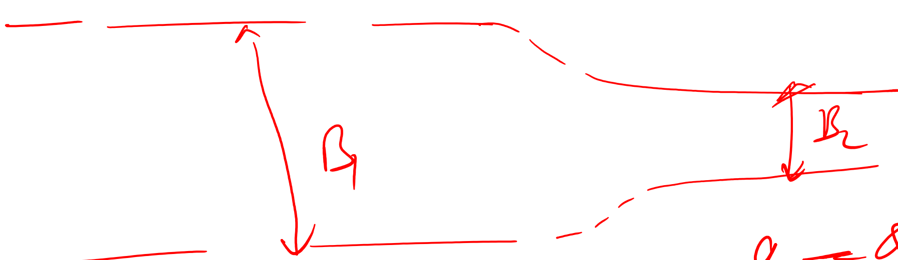
$y \Rightarrow E_c$ ✓

$E_2 = \square$

if $E_2 > E_c$ → case I

$E_2 = E_c$, case II

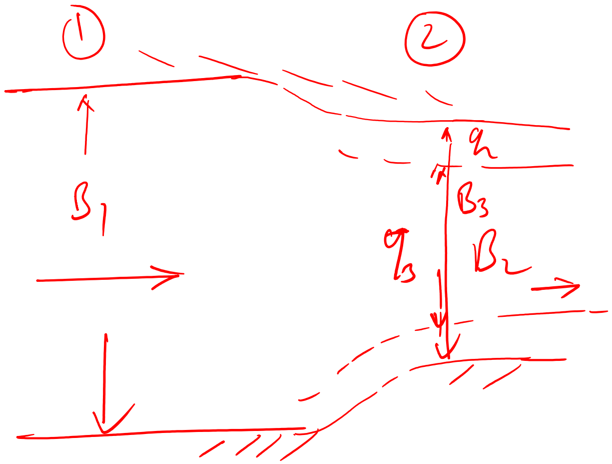
$E_c = E_m$ $\Delta Z > \Delta Z_m$

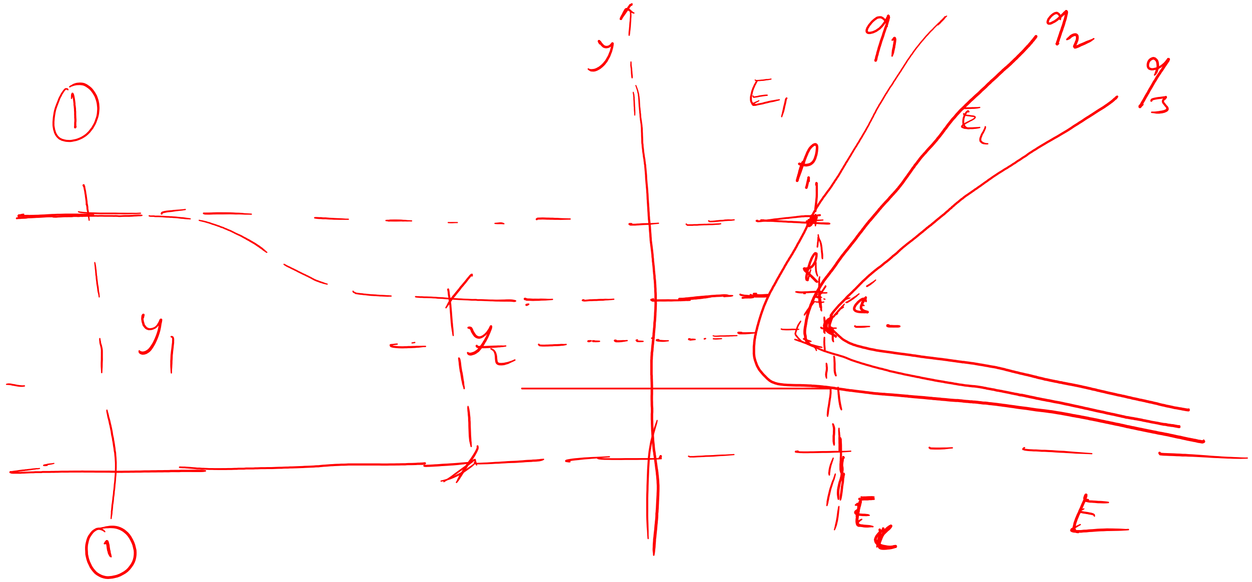


$q_1 = Q/B_1$ $q_2 = Q/B_2$

$q_2 > q_1$

$$\underbrace{y_1 + \frac{q_1^2}{2g y_1^2}}_{E_1} = \left(y_2 + \frac{q_2^2}{2g y_2^2}\right) \rightarrow E_2$$





$E_1 = E_2 = E_c$

Case I

$B_2 > B_m$

$E_1 = E_2$

$q_2 = Q/B_2$

$$y_1 + \frac{q_1^2}{2gy_1^2} = y_2 + \frac{q_2^2}{2gy_2^2}$$

$$y_1 + \frac{q_1^2}{2gy_1^2} = y_2 + \frac{(Q/B_2)^2}{2gy_2^2}$$

$$\Rightarrow B_2 = \square$$

Case II

$B_2 = B_m$

$E_1 = E_1 = E_c$

$y_1 + \frac{q_1^2}{2gy_1^2} = \frac{3}{2} y_c$

$\Rightarrow B_m = \boxed{}$ ✓

$\alpha\, y_c = \left(\frac{q_2^2}{g}\right)^{1/3}$

$= \left(\frac{Q^2}{B_m^2 g}\right)^{1/3}$

$q_2 \rightarrow y_c = \left(q_2^2/g\right)^{1/3}$

~~Case III~~

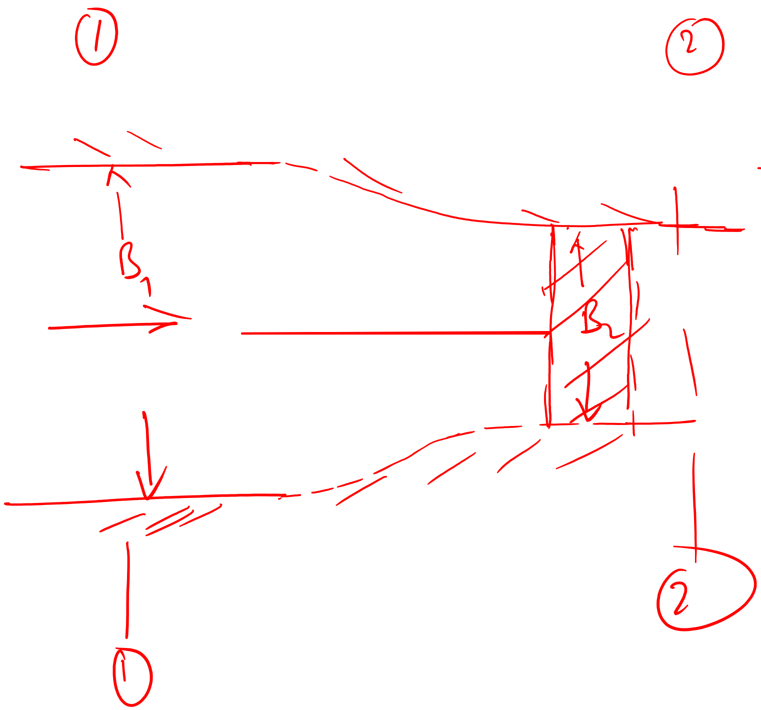
E_1'

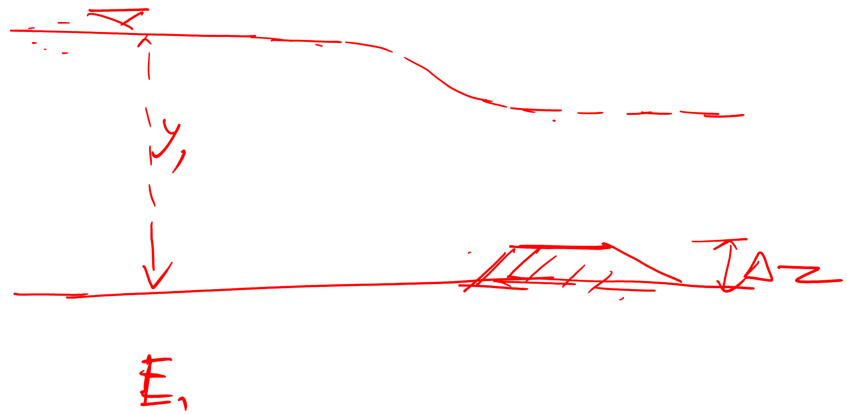
$\frac{3}{2} y_{c_2}$

$E_1' = E_{c_2} = \left(y_{c_2} + \frac{q^2}{2g y_{c_2}^2}\right)$

$y_1' + \frac{q_1^2}{2gy_1^2} =$

Hump + contraction





$y_c = \square$

$E_c = \square$

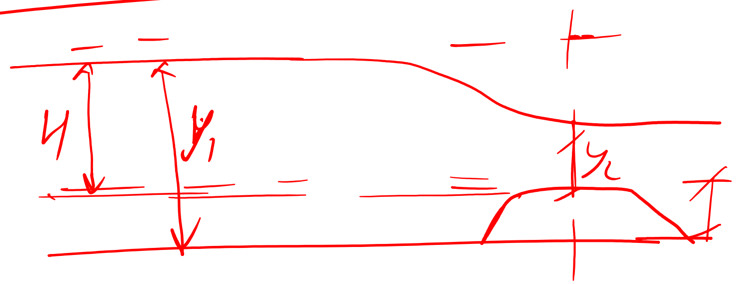
$$H_1 = E_1 = y_1 + \frac{q_1^2}{2g y_1^2}$$

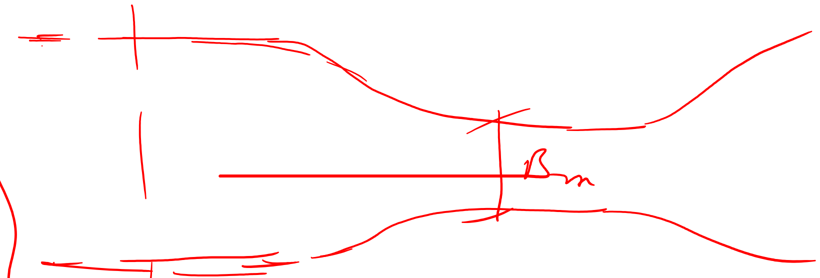
$$H_2 = E_2 + \Delta z = y_2 + \frac{q_2^2}{2g y_2^2} + \Delta z$$

$$E_1 = E_2 + \Delta z$$

Venturi flume

Broad crested weir





$$E_1 = E_c + \Delta Z$$

$$= y_c + \frac{q^2}{2g y_c^2} + \Delta Z$$

$$E_1 = E_c = \frac{3}{2} y_c$$

$$y_1 + \frac{v_1^2}{2g} = \frac{3}{2} y_c$$

$$y_1 + \frac{v_1^2}{2g} =$$

$$Q = \boxed{}$$

$$y_1 - \Delta Z + \frac{v_1^2}{2g} = \frac{3}{2} y_c \qquad q = Q/B$$

$$H = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} \Rightarrow Q = 0.544\sqrt{g}\, B H^{3/2}$$

① variation of Hydraulic radius R with depth of flow y & x-section Area A

② Momentum Equation

③ Specific Force

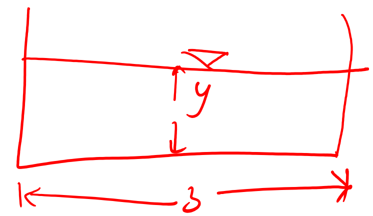
④ Specific force diagram

①

$$R = A/P$$

$$A = by$$

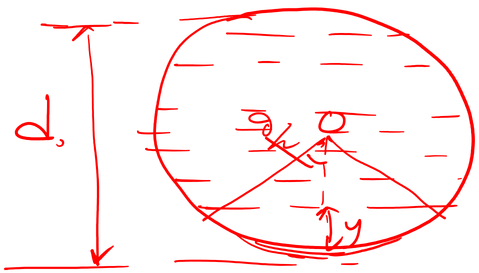
$$P = b + 2y$$



$\boxed{A, b, y, R, T, , K, Z}$

R vs y → , R vs A
R vs K
R vs Z

circular channel.



$$R = A/P$$

$$A = \frac{d^2}{8}(\theta - \sin\theta)$$

$$P = D\theta/2$$

$$R = \frac{d}{4\theta}(\theta - \sin\theta)$$

1 m

y

$$\cos\theta/2 = \left(1 - \frac{2y}{D}\right)$$

$$\theta = 2\cos^{-1}(1 - 2y/D)$$

y/d_o

R/R_o

y	θ	R
0.1		
0.2		
0.3		
1.0		

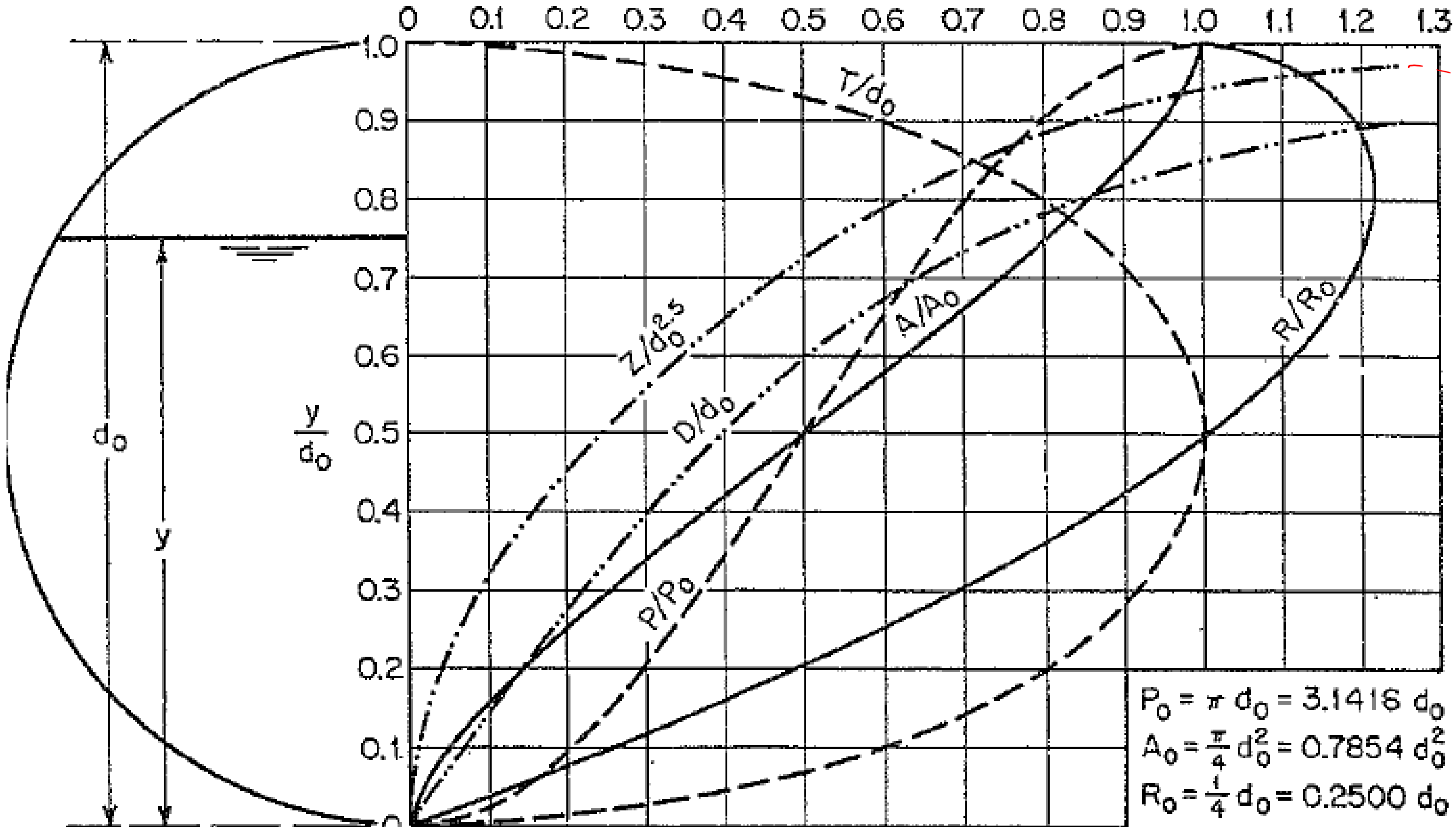
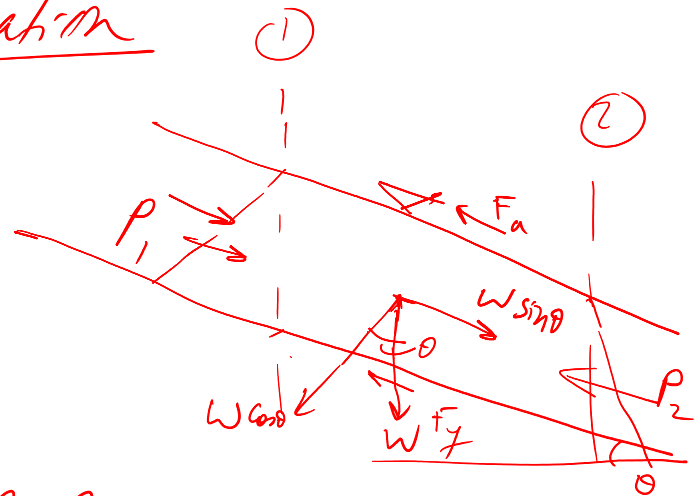
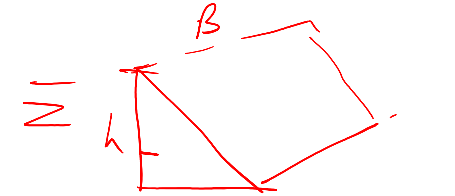


FIG. 2-1. Geometric elements of a circular section.

Momentum Equation



$P = \rho g h$



$P = \rho g h \times B \times \bar{Z}$

$= \rho g A \bar{Z}$

$= \gamma A \bar{Z}$

Net force acting on a control volume = change in momentum

$P = \rho g h$

or $\sum F_x = m_2 - m_1$

or) $W\sin\theta - F_f - F_a + P_1 - P_2 = \rho Q (u_2 - u_1)$

or) $W\sin\theta - F_f - F_a + P_1 + \rho Q u_1 = P_2 + \rho Q u_2$

or) $W\sin\theta - F_f - F_a + \gamma A_1 \bar{Z} + \rho Q \cdot \frac{Q}{A_1} = \gamma A_2 \bar{Z} + \rho Q \cdot \frac{Q}{A_2}$

a) $\frac{W\sin\theta - F_f - F_a}{\rho g} + A_1 \bar{Z} + \frac{Q}{g}\frac{Q^2}{A_1} = A_2 \bar{Z} + \frac{Q^2}{g A_2}$

$$\left(\frac{W\sin\theta - F_f - F_a}{\gamma g}\right) + \left(A_1\bar{z}_1 + \frac{Q^2}{A_1 g}\right) = \left(A_2\bar{z}_2 + \frac{Q^2}{A_2 g}\right)$$

$$\left(A_1\bar{z}_1 + \frac{Q^2}{A_1 g}\right) \Rightarrow$$ its unit is equals to Force per unit weight of fluid.

$\Downarrow$

Force per unit weight of fluid.

$$\frac{F}{\gamma} = \frac{kg \cdot m/s^2}{kg/m^3 \times m/s^2}$$

$F = ma$

$$= \boxed{m^3}$$

This term is known as specific force

$$\left(\frac{W\sin\theta - F_f - F_a}{\delta g}\right) + \left(A_1\bar{Z}_1 + \frac{Q^2}{A_1 g}\right) = \left(A_2\bar{Z}_2 + \frac{Q^2}{A_2 g}\right)$$

if we we consider small stretch element.

$\theta \approx 0$, $\therefore \sin\theta = 0$

$\therefore F_a \approx 0$

$F_f \approx 0$

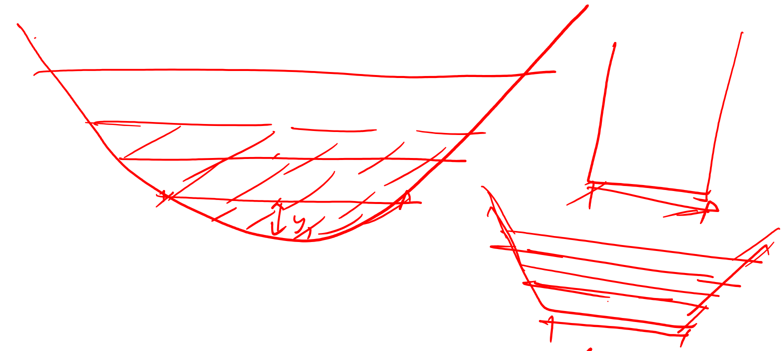
Then we can write above eqn as

$$A_1\bar{Z}_1 + \frac{Q^2}{A_1 g} = A_2\bar{Z}_2 + \frac{Q^2}{A_2 g}$$

E

$$\boxed{M_1 = M_2}$$

Specific force diagram



$$M = A_1 \bar{Z}_1 + \frac{Q^2}{A_1 g}$$

for given Q, $M = f(y)$

now we wants to know the condn when Sp. Force is minm.

for minm. Sp force $\frac{dM}{dy} = 0$

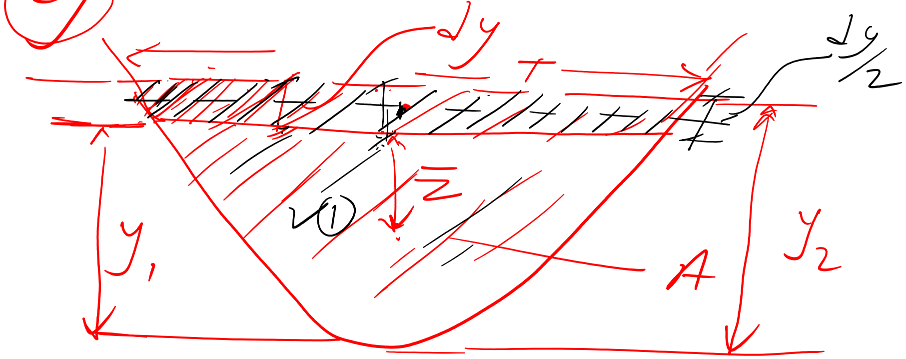
$$\frac{dM}{dy} = \frac{Q^2}{g}\left(\frac{dA^{-1}}{dy}\right) + \frac{d(A\bar{Z})}{dy}$$

$$= \frac{Q^2}{g}\left(-1\, A^{-2} \frac{dA}{dy}\right) + \frac{d(A\bar{Z})}{dy}$$

$$\frac{dM}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d(A\bar{z})}{dy}$$

→ moment of

$A\bar{z}$ = first moment of Area from water surface,



$$y_2 - y_1 = \Delta y$$

$$\frac{dM}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{\left(A\left(\bar{z} + dy\right) + T\,dy \cdot \frac{dy}{2}\right) - A\bar{z}}{dy}$$

$$\frac{dM}{dy} = \frac{-Q^2}{gA^2}\frac{dA}{dy} + \frac{\left(A\,dy + T\frac{(dy)^2}{2}\right)}{dy}$$

neglecting $\frac{(dy)^2}{2}$

$$\frac{dM}{dy} = \frac{-Q^2}{gA^2}\frac{dA}{dy} + \frac{A\,dy}{dy}$$

$~~T = dA~~$ $T = \frac{A}{y}$

$$\boxed{T = \frac{dA}{dy}}$$

$$\frac{dM}{dy} = -\frac{Q^2 T}{gA^2} + A$$

now for minm. M, $\frac{dM}{dy} = 0$

$\frac{dE}{dy} = 0$

so $-\frac{Q^2 T}{gA^2} = -A$

or $\boxed{\frac{Q^2 T}{gA^3} = 1}$ $\rightarrow$ $\boxed{\frac{V}{\sqrt{gy}} = 1}$ =

so this is the condn. for critical flow,

$$M = A\bar{Z} + \frac{Q^2}{Ag}$$

$$\frac{Q^2 T}{gA^3} = 1 \quad \text{(critical flow)}$$

$$\Downarrow$$

or) $$Q = \sqrt{gA}\,\sqrt{\frac{M - A\bar{Z}}{y}}$$

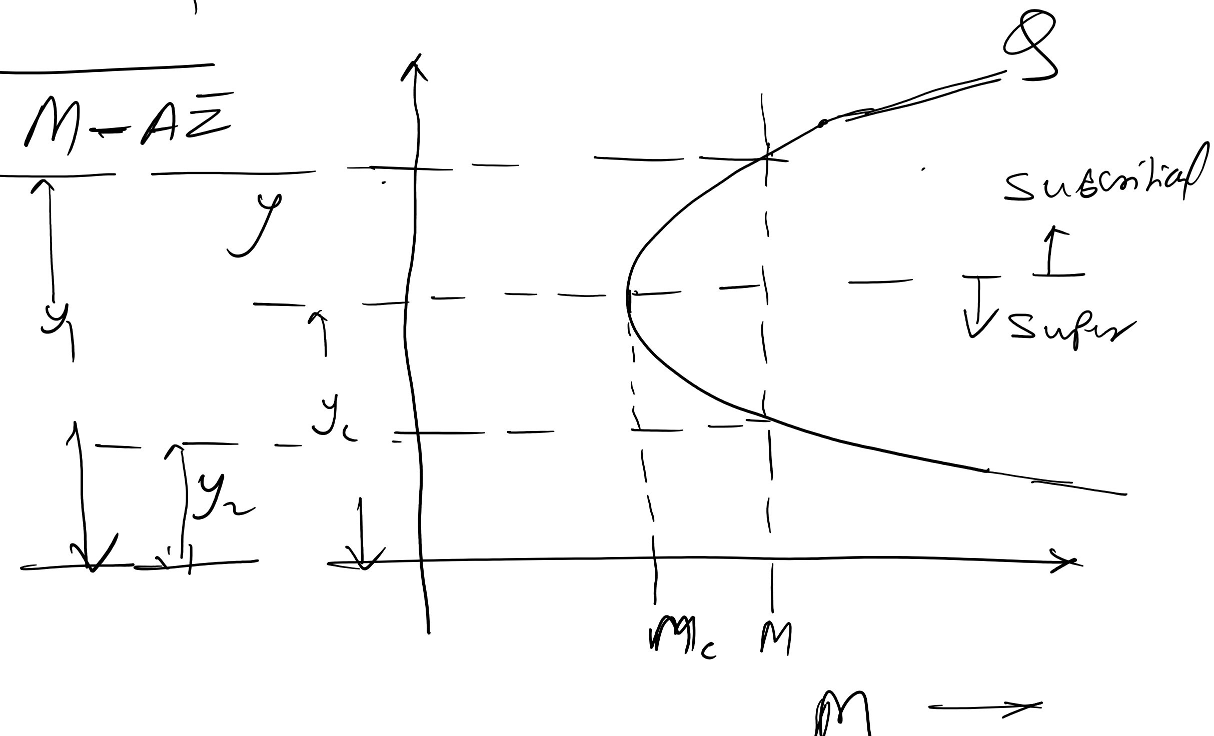


Fig: Sp. Force diagram

for given specific force of a channel discharge is maximum at critical flow condn

$$\frac{dQ}{dy} =$$

$$Q = \left(\sqrt{gA}\ \sqrt{M - A\bar{z}}\right) \quad \text{--- (1)}$$

for maxm. discharge for given Specific energy that channel can pass, $M = A\bar{z} + \frac{Q^2}{gA}$

$$\frac{dQ}{dy} = 0$$

$$\Rightarrow \frac{Q^2 T}{gA^3} = 1 \Rightarrow \text{The flow is critical}$$

× Critical Depth condn.

(i) Sp. energy is minm for a given discharge

(ii) Discharge is maxm at critical flow for given Sp. energy

(iii) Sp. force is minm. for a given discharge.

iv) Discharge is maxm. for a given Sp. force.

v) Froude number is unity