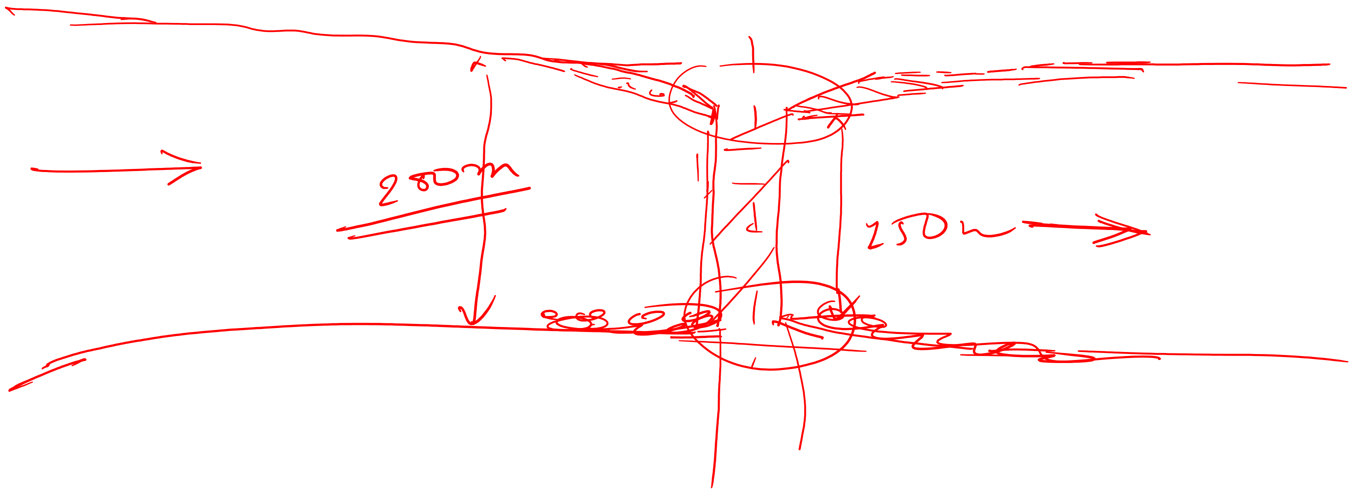
Channel Transition

→ contraction → canals

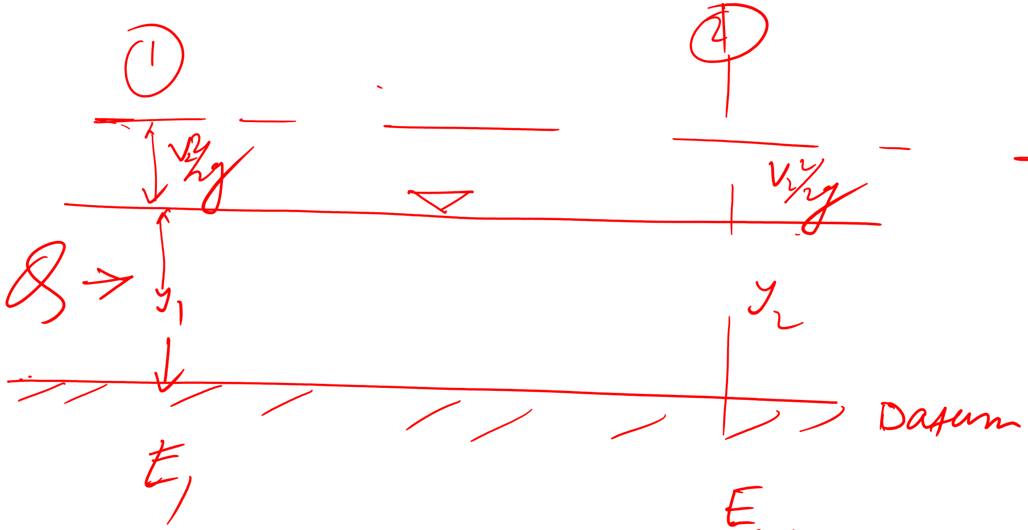
→ rise / fall in channel bed level.

U/S



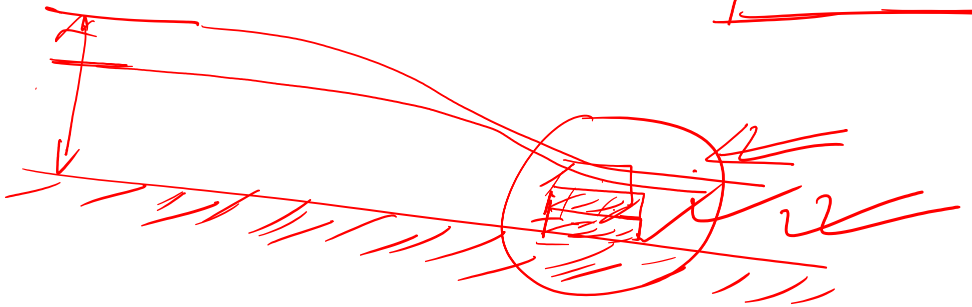
(Hump → weir)

channel transition with hump (rise in bed level)



$$E_1 = E_2$$

$$\boxed{E_1' > E_1}$$

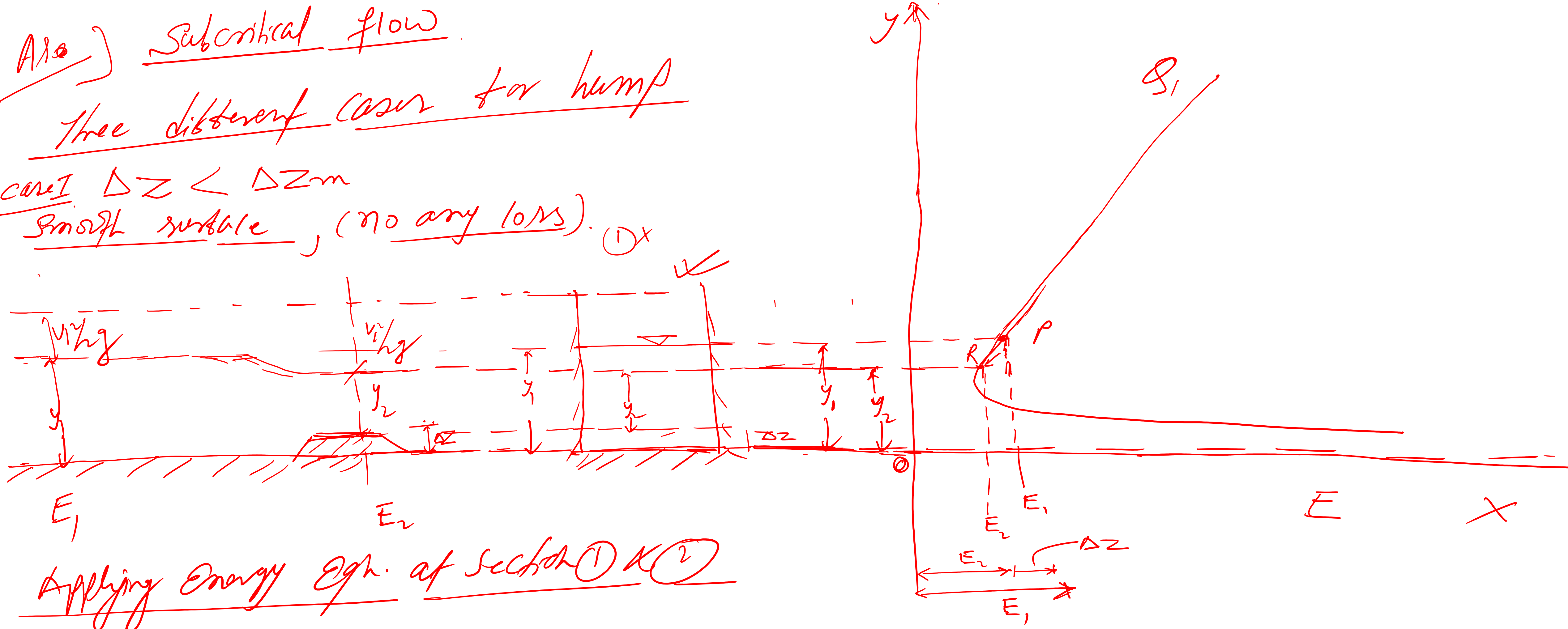


A) Subcritical flow

Three different cases for hump

case I $\Delta Z < \Delta Z_m$

Smooth surface, (no any loss).



Applying Energy Eqn. at section 1 & 2

$$H_1 = H_2$$

$$\Rightarrow E_1 = E_2 + \Delta Z$$

$$E_2 = E_1 - \Delta Z$$ → So energy at section 2 drops by ΔZ

$$E_1 = \Delta Z + E_2$$

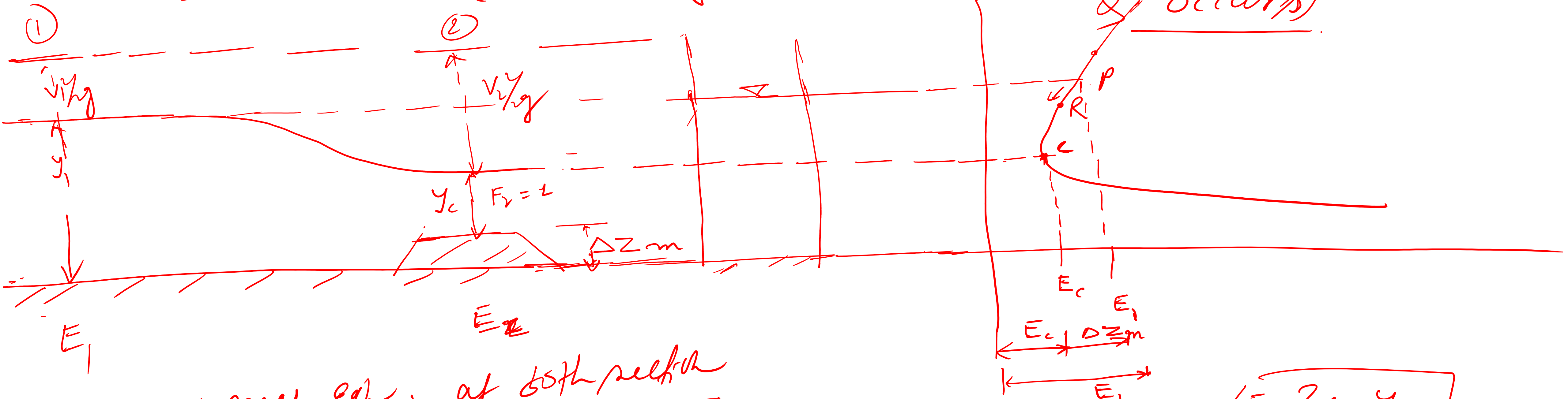
$$\boxed{V = \frac{1}{n} R^{2/3} S^{1/2}}$$

$$\Rightarrow y_1 + \frac{V_1^2}{2g} = \Delta Z + y_2 + V_2^2/2g$$

$$\Rightarrow y_1 + \frac{Q^2}{2gA_1^2} = \Delta Z + y_2 + \frac{Q^2}{2gA_2^2}$$

Case II

$\Delta Z = \Delta Z_m$ (Minm. height of hump at which critical depth occurs)



Applying energy eqn. at both section

$$E_1 = E_2 + \Delta Z_m$$

$$E_1 = E_c + \Delta Z_m = \Delta Z_m + E_c$$

$$= \Delta Z_m + y_c + \frac{q^2}{2g y_c^2}$$

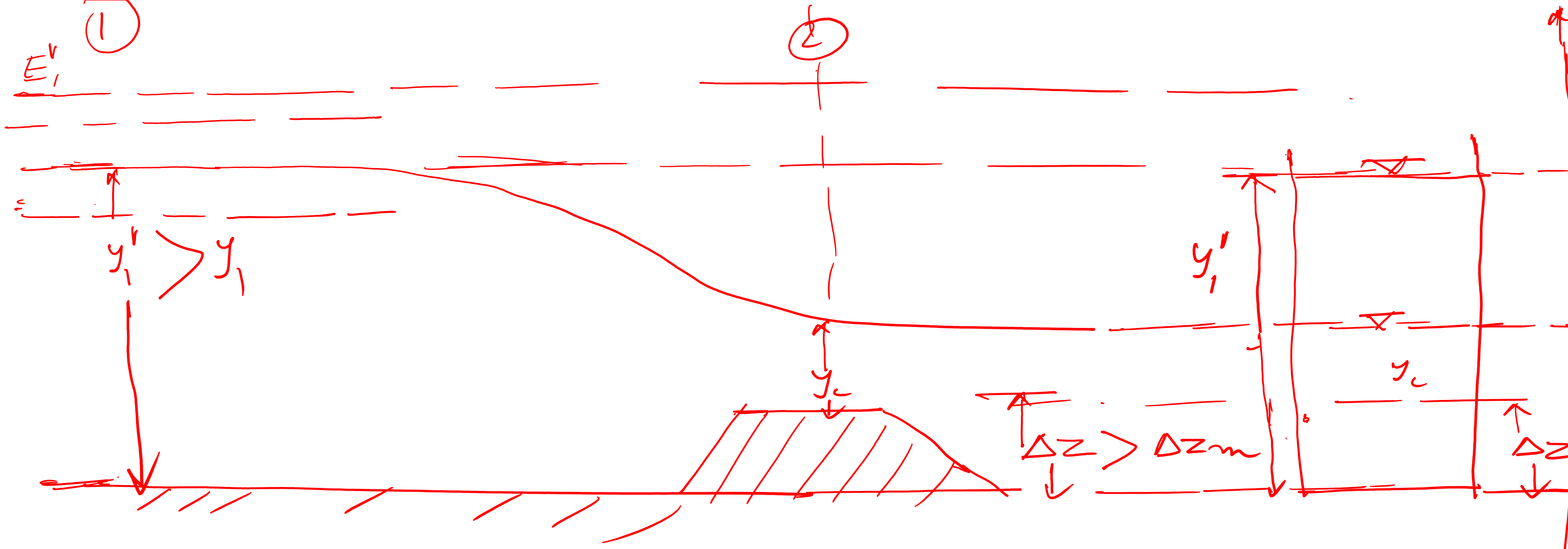
$$\boxed{E_c = \frac{3}{2} y_c}$$

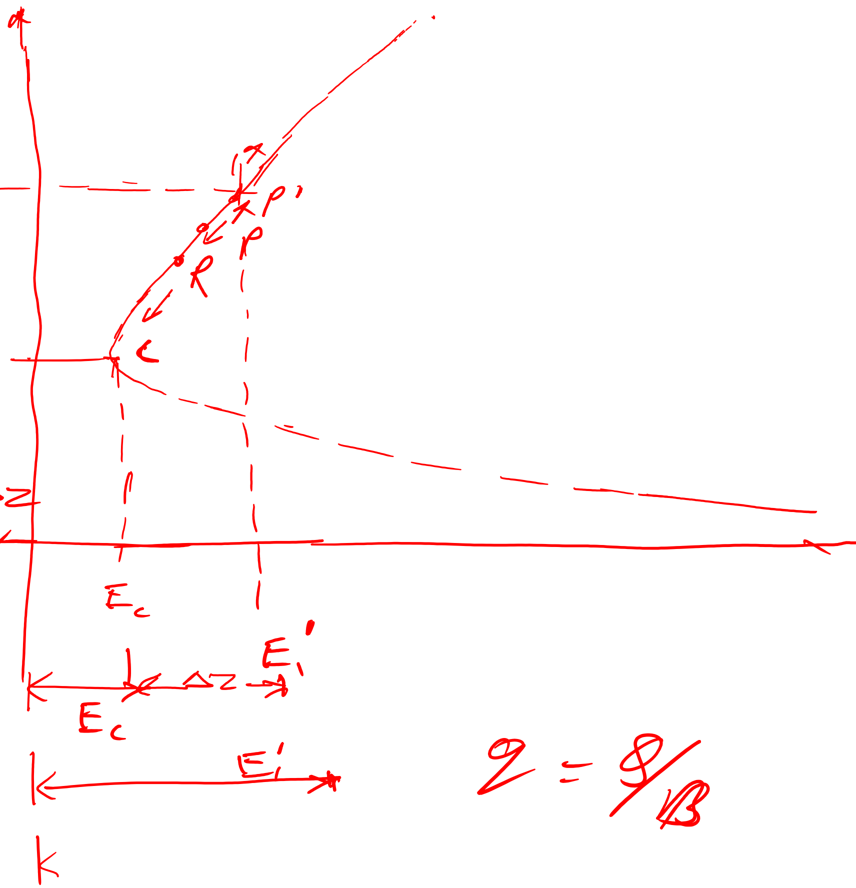
$$\boxed{E_1 = \Delta Z_m + \frac{3}{2} y_c}$$

Case III ($\Delta Z > \Delta Z_m$)

now increase the height of hump such that $\Delta Z > \Delta Z_m$, at this condition, the water depth at section (2) still remains critical, y_c however at section (1) the flow has not sufficient energy to pass down stream and thus it increases its depth to y_1', consequently Energy level to new Energy level E_1' such that $E_1' > E_1$

contd





$q = Q/B$

Applying energy eqn at section 1 & 2

$$E_1' = E_c + \Delta Z$$

$$= y_c + \frac{q^2}{2g y_c^2} + \Delta Z$$

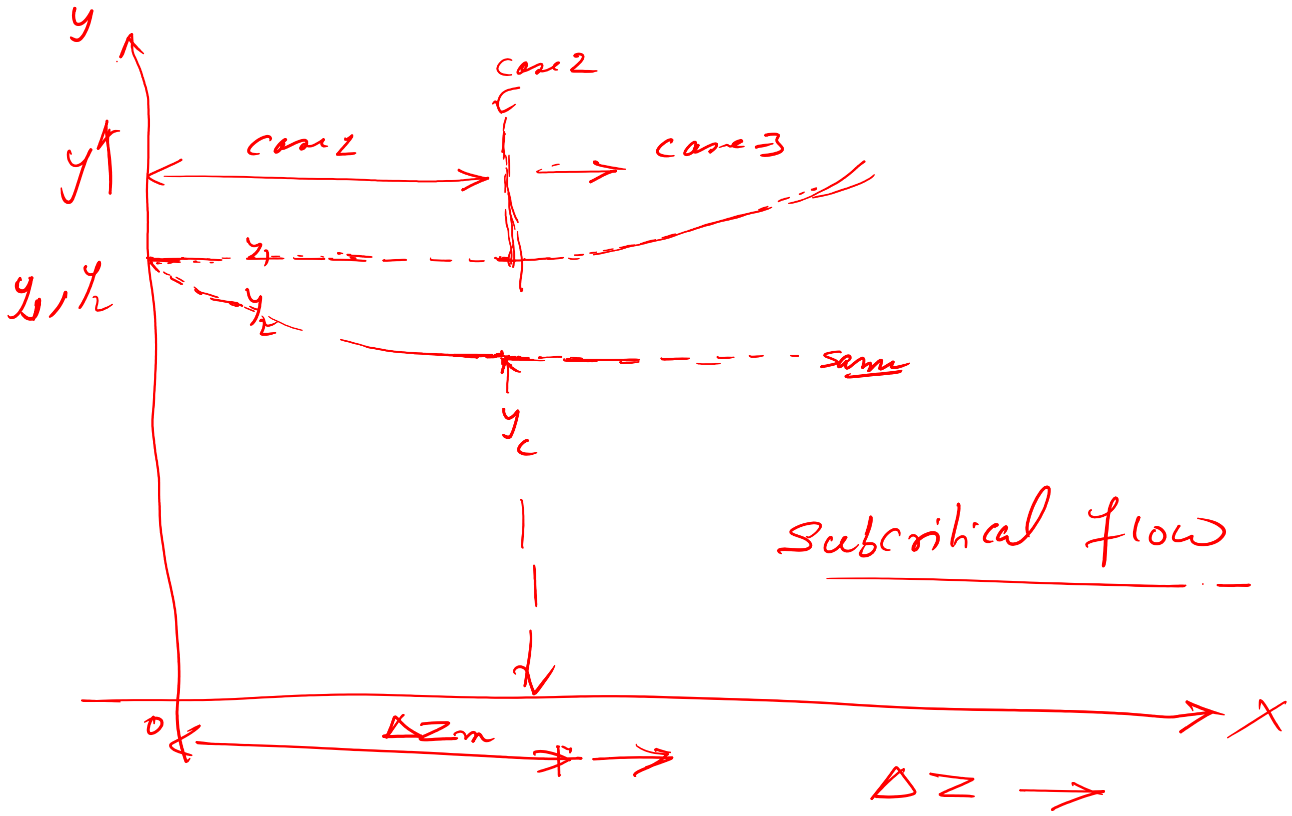
$$E_1' = \frac{3}{2} y_c + \Delta Z$$

$$y_1' + \frac{q^2}{2g y_1'^2} = \frac{3}{2} y_c + \Delta Z$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

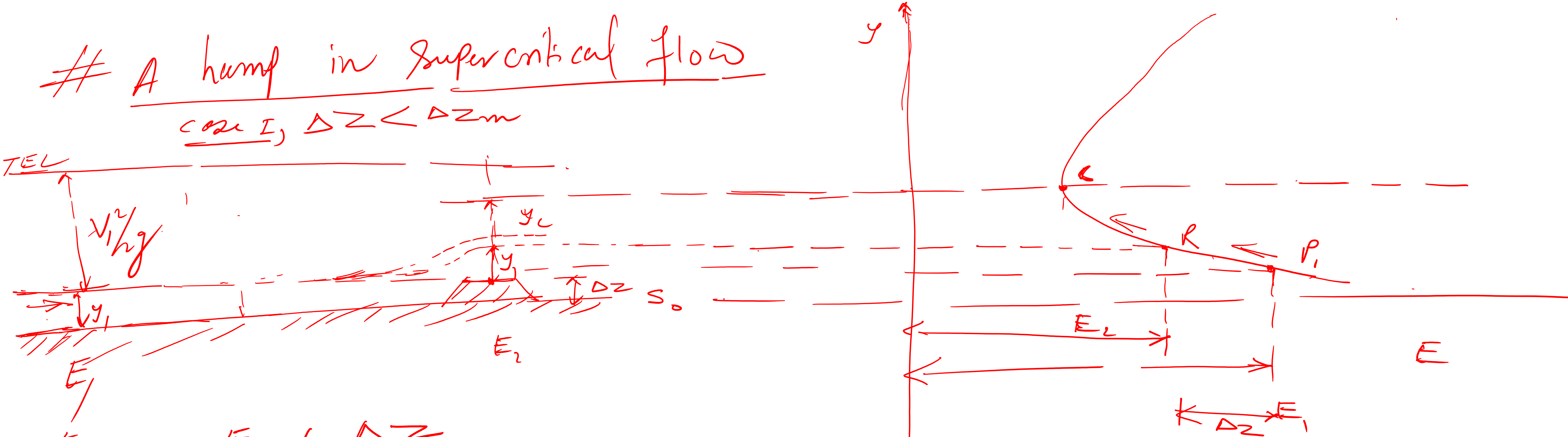
Summary of all 3 cases in a graph

i.e., variation of y_1, y_2 with hump height ΔZ.



A hump in Supercritical flow

Case I, $\Delta Z < \Delta Z_m$



$$E_1 = E_2 + \Delta Z$$

Case II

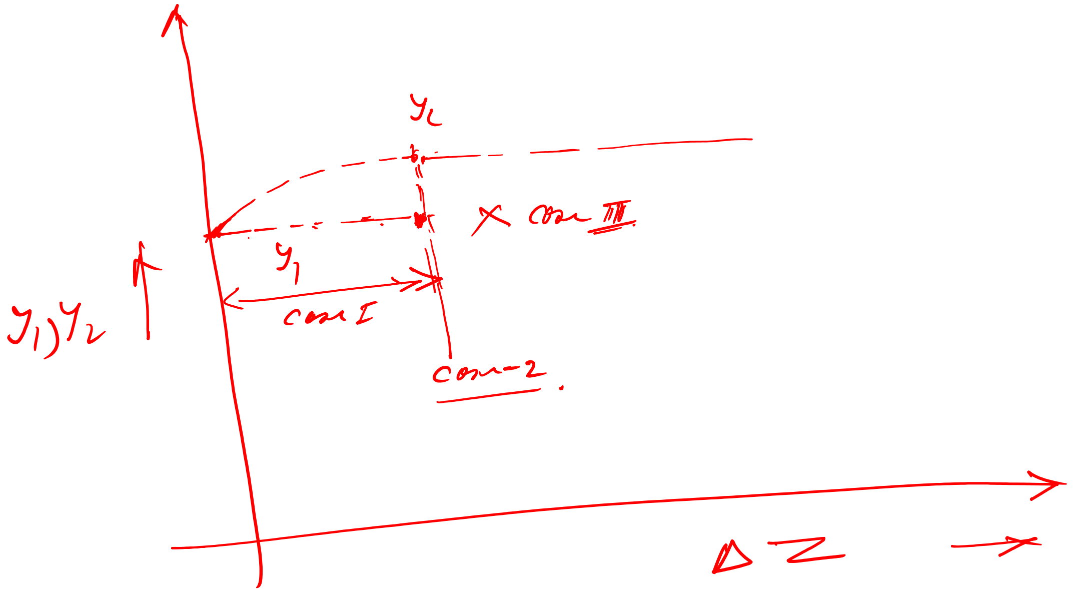
$$\Delta Z = \Delta Z_m$$

$$E_1 = E_c + \Delta Z_m$$

Case III, $\Delta Z > \Delta Z_m$

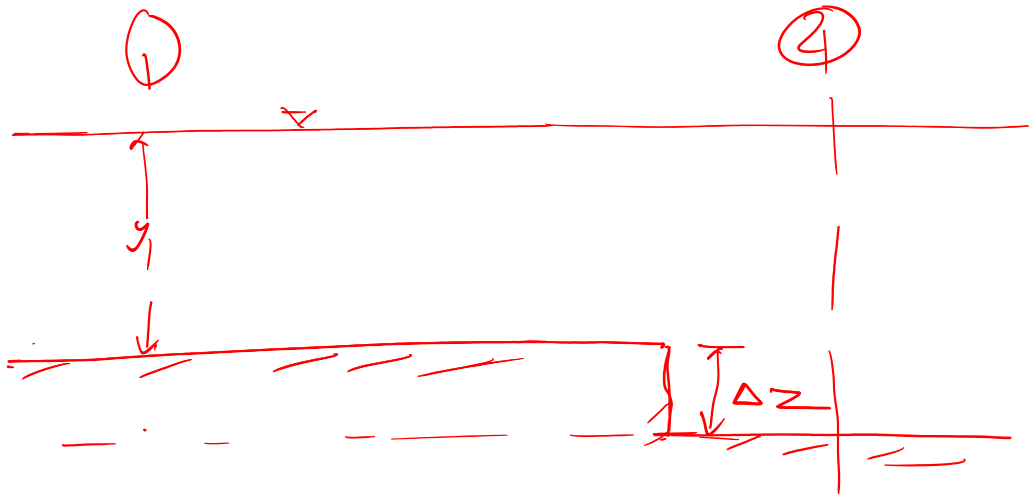
flow doesn't remains supercritical, there will be formation of Hydraulic Jump u/s of Hump

Summary



Supercritical flow

* Drop in bed level.



$E_1 = E_2 + \Delta z$

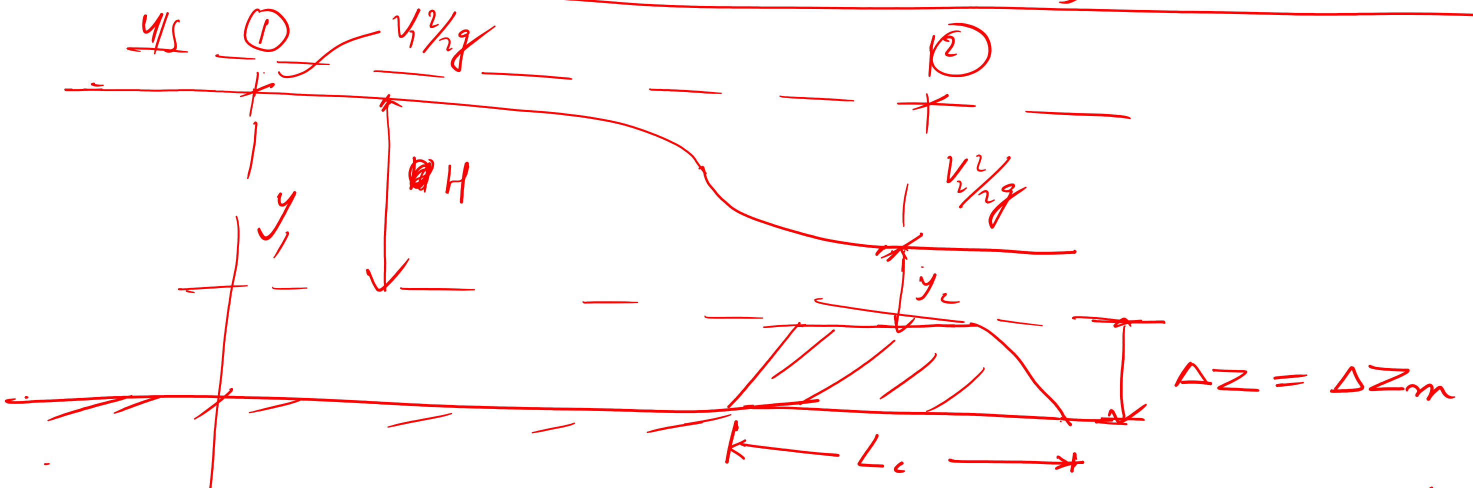
$$\boxed{E_1 = E_2 - \Delta z}$$

$$\boxed{\Delta z + \left(y_1 + \frac{V_1^2}{2g}\right)} = E_2$$

$\Delta z + E_1 = E_2$

$E_1 = E_2 - \Delta z$

* Application of Energy eqn. (concept of Hump) for Broad crested weir.



Now applying the energy at section 1 & 2

$$\frac{L_c}{(y_1 - \Delta Z)} > 1.5 - 3$$

$$E_1 = E_2 + \Delta Z$$

$$\therefore \frac{L_c}{H} > 1.5 - 3$$

$$E_1 = E_c + \Delta Z$$

$$\Rightarrow \left(y_1 + V_1^2/2g\right) = \frac{3}{2} y_c + \Delta Z$$

$\Rightarrow$ $\because$ $V_1^2/2g << y_1$, so we neglect velocity head,

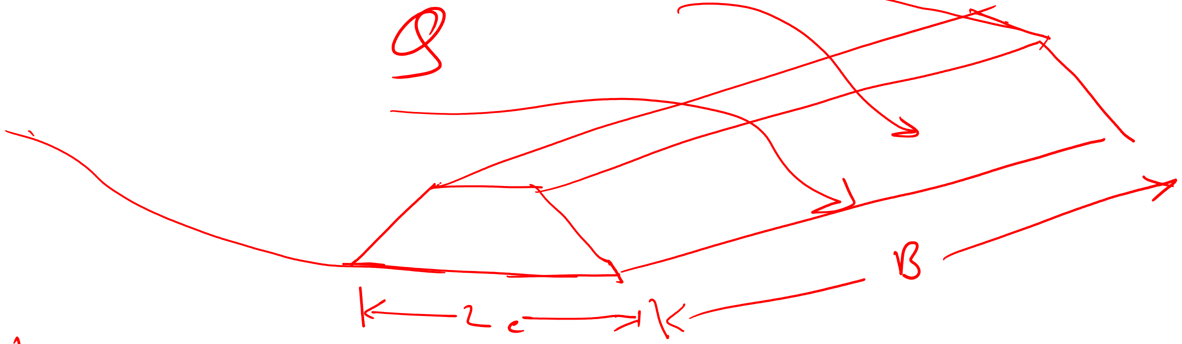
$\Rightarrow y_1 = \frac{3}{2} y_c + \Delta Z$

$y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$\Rightarrow (y_1 - \Delta Z) = \frac{3}{2} y_c$

$\Rightarrow H = \frac{3}{2} y_c$

$q = Q/B$



$\Rightarrow H = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$

$\Rightarrow H^3 = \left(\frac{3}{2}\right)^3 \left(\frac{(Q^2/B^2)}{g}\right)$

$\Rightarrow H^3 \times g \times \left(\frac{3}{2}\right)^3 = Q^2/B^2$

$\Rightarrow$

or)

$$Q^2 = \left(\frac{2}{3}\right)^3 * H^3 * B^2 * g$$

$$\boxed{Q = \left(\frac{2}{3}\right)^{3/2} \sqrt{g}\; B\, H^{3/2}} \longrightarrow \text{Discharge eqn of}$$

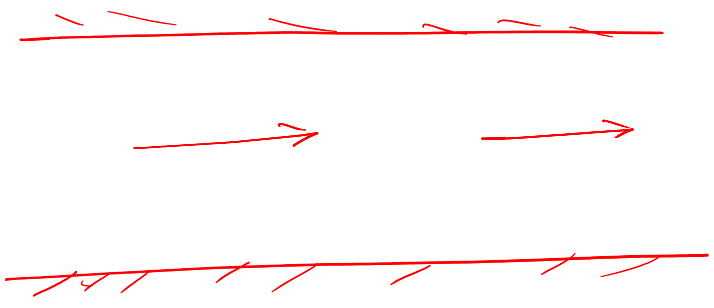
Broad crested weir

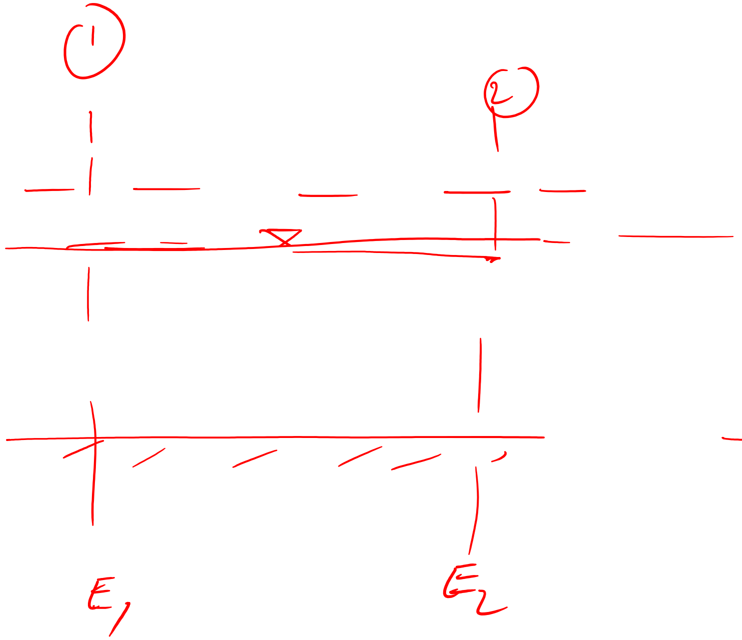
Channel contraction

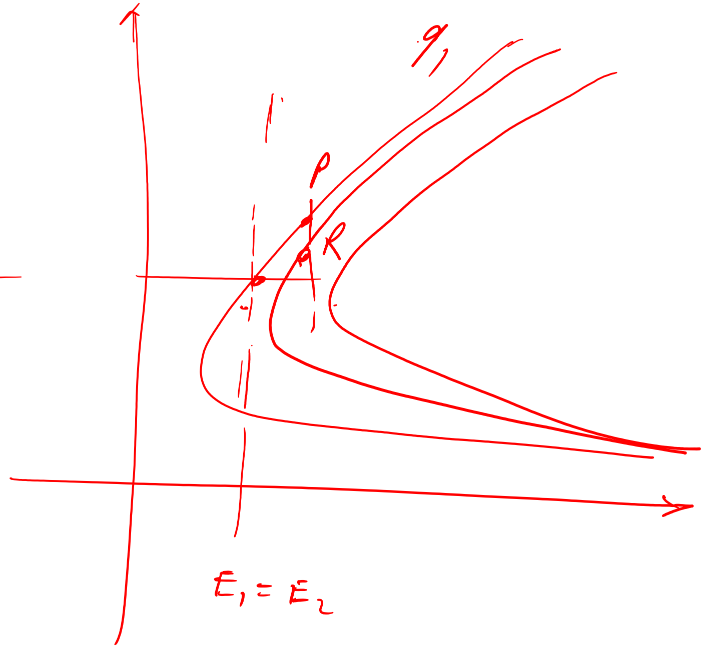
Application:

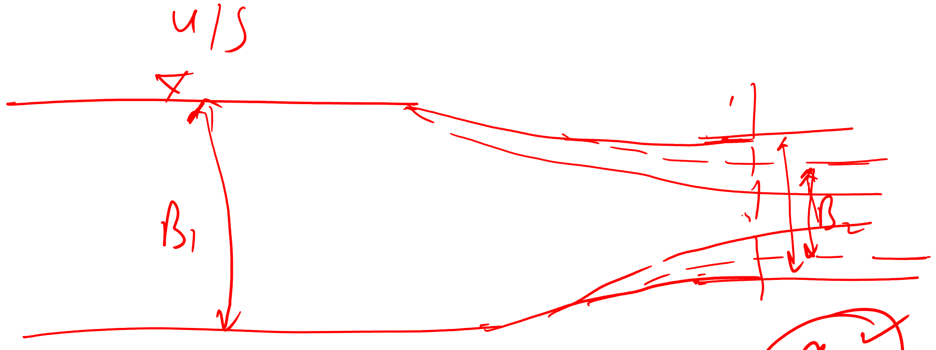
1. to connect two different width of channel, without any appreciable loss in energy
2. contraction is necessary in river crossing structures
3. As a discharge measuring structure or device. (venturi flume)

channel contraction









$B_2 < B_1$

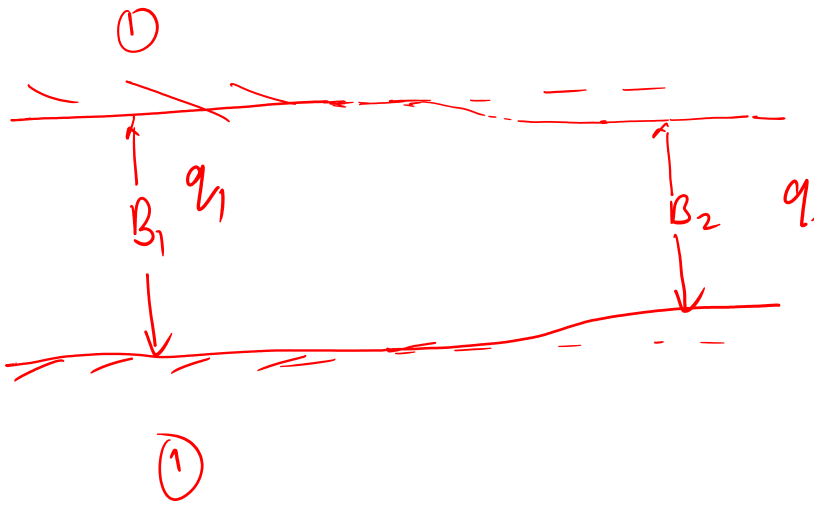
$q_1 = Q/B_1$

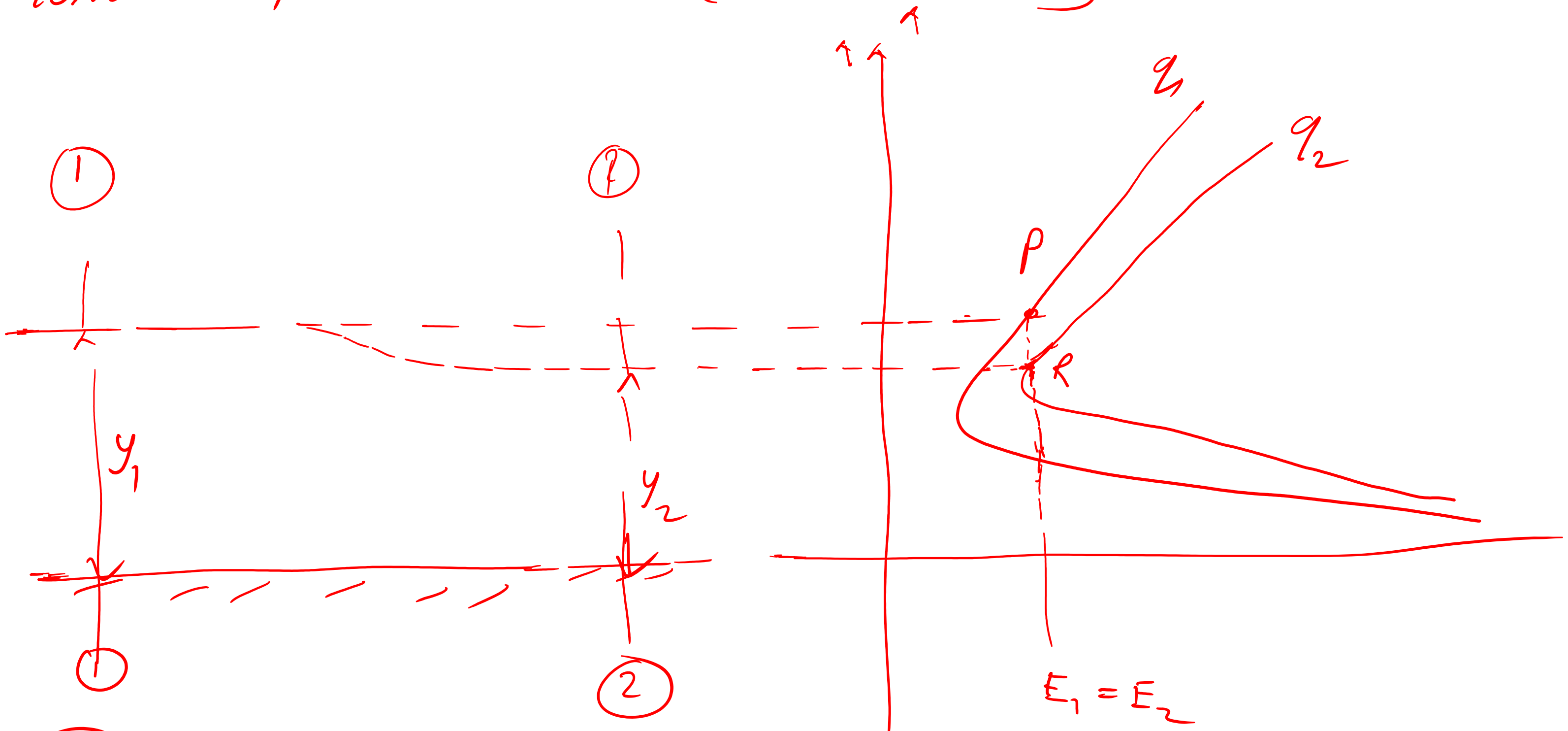
$q_2 = Q/B_2$, $q_2 > q_1$, $q_3 > q_2$

Channel contraction

Assumption: there is no any losses at contraction (Section 2-2)

Case I





Applying energy eqn at ① and ②

$$E_1 = E_2$$

$$\Rightarrow y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{q_1^2}{2g y_1^2} = y_2 + \frac{q_2^2}{2g y_2^2}$$

$q_1 = Q/B_1$

$q_2 = Q/B_2$

$Q = B_1 y_1 * V_1$

$\frac{Q}{B_1} = y_1 V_1$

$q_1 = y_1 V_1$

$\Rightarrow V_1 = q_1/y_1$

$$y_1 + \frac{v_1^2}{2g} = y_2 + V_2^2/2g$$

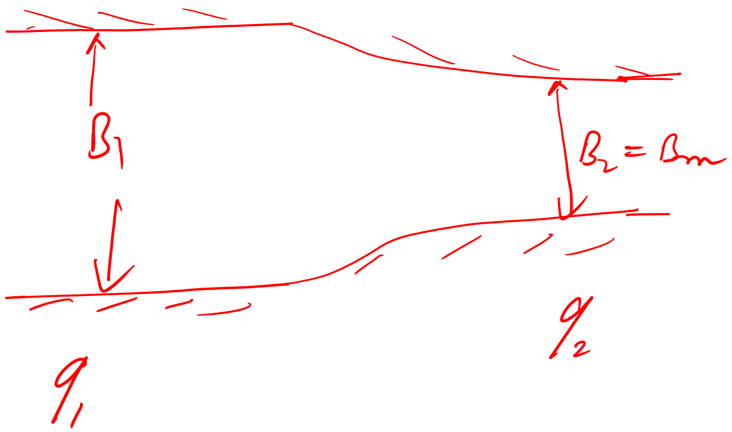
$$y_1 + \frac{Q^2}{2g(B_1 y_1)^2} = y_2 + \frac{Q^2}{2g(B_2 y_2)^2}$$

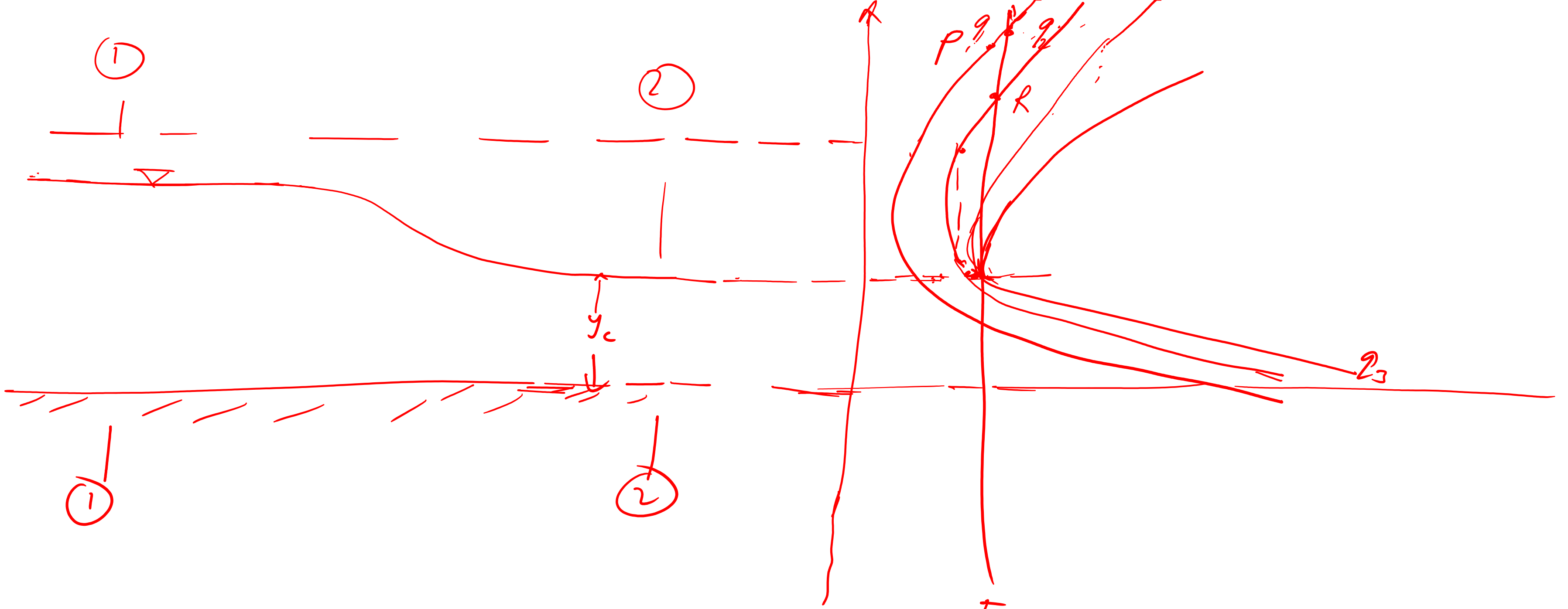
$$B_2 = \boxed{} \quad \cancel{2}$$

Case II

now reducing channel width at section 2-2, such that there will be critical flow

$B_2 = B_m$, minm. ~~width of~~ width of channel at section 2-2, at which flow becomes critical.





$E_1 = E_c$

Applying Energy Eqn

$$E_1 = E_2$$

$$y_1 + \frac{Q^2}{2gA_1^2} = E_c$$

$$= y_c + \frac{Q^2}{2gA_c^2}$$

$$y_1 + \frac{Q^2}{2g(B_1 y_1)^2} = y_c + \frac{Q^2}{2g \times (B_m \times y_c)^2} \Rightarrow B_m = \boxed{}$$

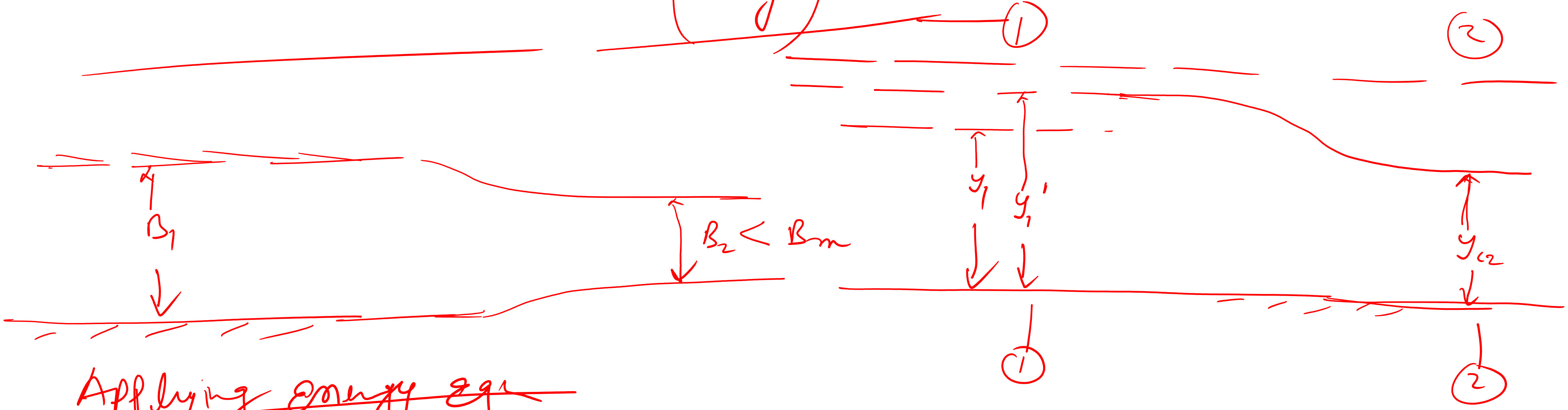
Case III

$B_2 < B_m$

$q_2 \rightarrow$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{q_{2m}}{g}\right)^{2}$$



Applying energy eqn

$$E_1' = E_2$$

$$y_1' + V_1^2/2g = y_{c_2} + V_2^2/2g$$

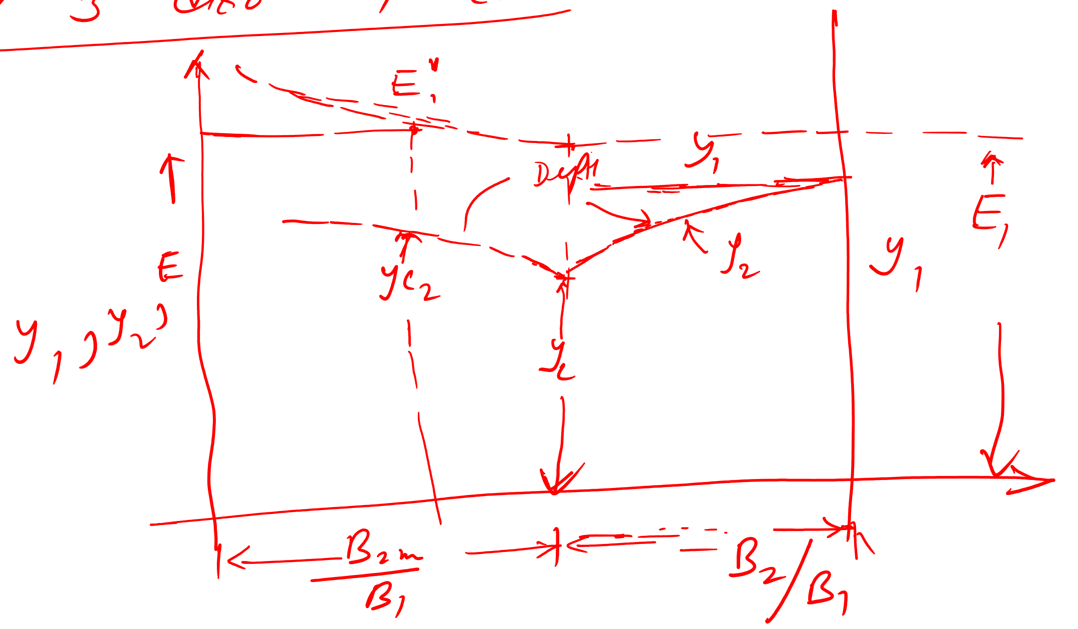
$$y_1' + \frac{Q^2}{2g(B_1 y_1')^2} = y_{c_2} + \frac{Q^2}{2g(B_2 y_{c_2})^2} \longrightarrow (1)$$

$$y_1' + \frac{Q^2}{2g(B_1 y_1')^2} = \frac{3}{2} y_{c_2}$$

$$E_c = 3/2\, y_{c_2}$$

$$y_{c_2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{(Q/B_2)^2}{g}\right)^{1/3}$$

* Summary of all 3 different cases.



* Choking

$\Delta Z = \Delta Z_m$

$\Delta Z \leq \Delta Z_m$

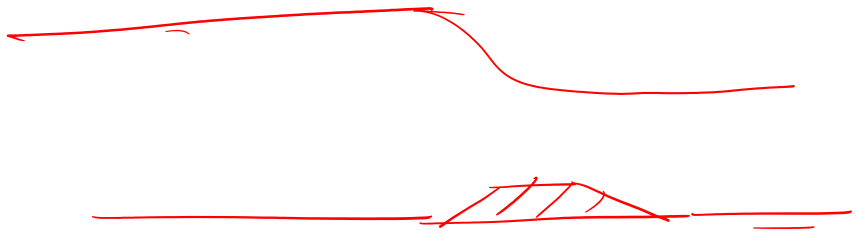
y_1 = constant
for all $\Delta Z \leq \Delta Z_m$

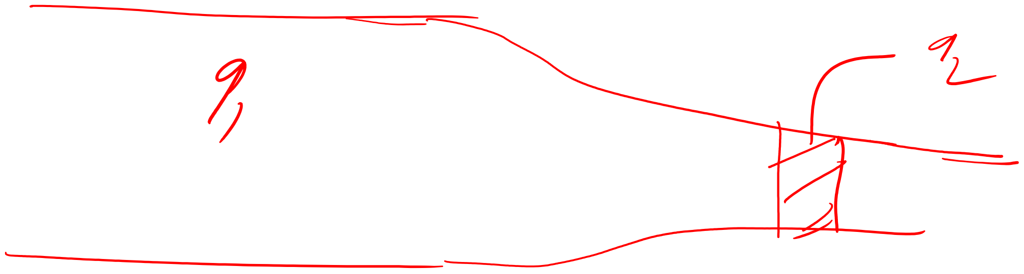
case III, choking

$B_2 < B_m$

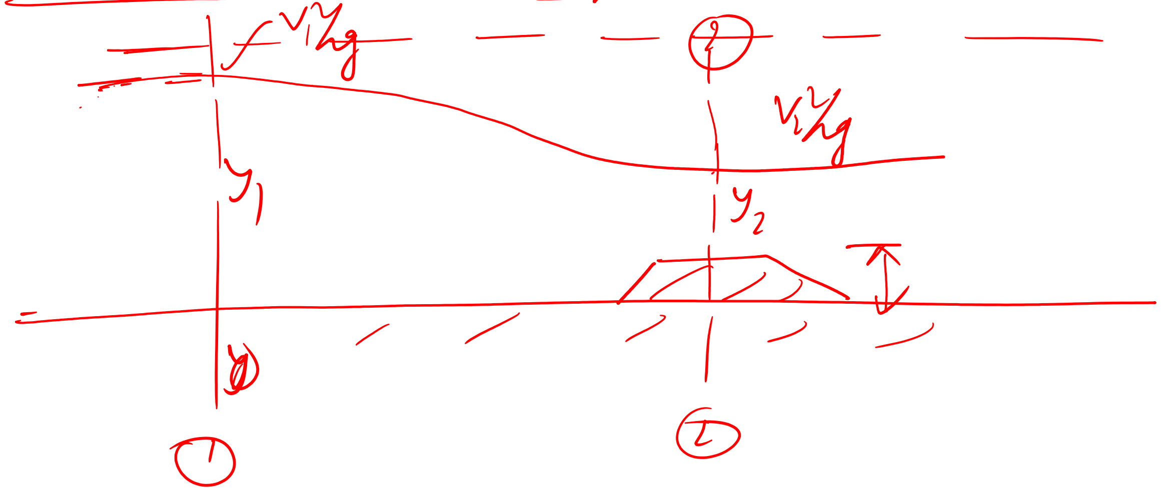
y_1'

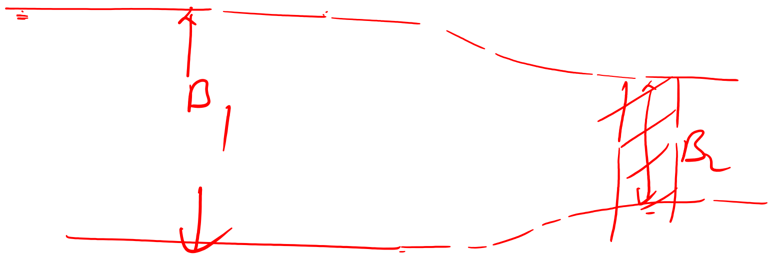
$\Delta Z > \Delta Z_m$





Channel with hump and contraction





$$E_1 = E_2 + \Delta Z$$

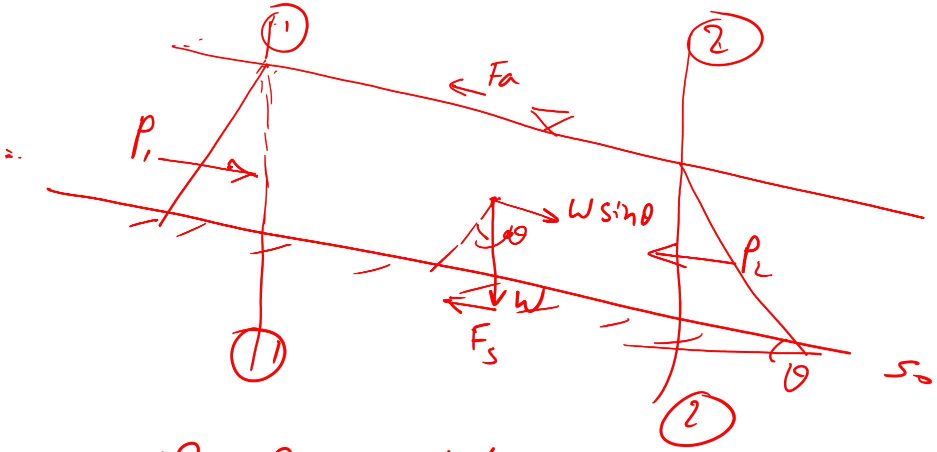
$$y_1 + \frac{Q^2}{2g(B_1^2 y_1^2)} = \Delta Z + y_2 + \frac{Q^2}{2g(B_2^2 y_2^2)}$$

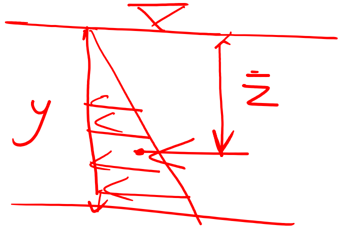
- momentum principle
- specific force and sp. force diagram
- criteria for critical state of flow, conjugate depth
- computer program coding for simple problem

momentum principle

$$\boxed{\sum F_x = \overline{m_2 - m_1}}$$

Let us consider a channel having bed slope S_0, x-section area A_1 and A_2 at two different section, Q is the discharge which passes through it,





$P_1 = \gamma y$

$\boxed{A = y * B}$

$\boxed{p = P/A}$

$\cancel{p = (\gamma \bar{z})}$

$P = p * A$
$= (\gamma \bar{z}) A$

consider a control volume betn. section 1 & 2

the forces acting on CV are: P_1, P_2, F_s, F_a, $W\sin\theta$

applying momentum principle,

$$\Sigma F_x = m_2 - m_1$$

$$\Rightarrow \quad P_1 - P_2 - F_s - F_a + W\sin\theta = \rho Q (u_2 - u_1)$$

$$\Rightarrow \quad (W\sin\theta - F_a - F_s) + \rho Q u_1 + P_1 = P_2 + \rho Q u_2$$

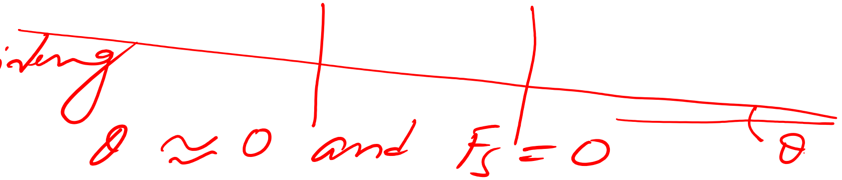
$$\Rightarrow \quad (W\sin\theta - F_a - F_s) + \rho Q u_1 + \gamma A_1 \bar{z}_1 = \gamma A_2 \bar{z}_2 + \rho Q u_2$$

$\Rightarrow$ Put $\gamma = \rho g$

$\Rightarrow$ $W\sin\theta - F_a - F_s + \rho g A_1 \bar{z}_1 + \rho Q u_1 = \rho g A_2 \bar{z}_2 + \rho Q u_2$

$$\Rightarrow \left(\frac{W\sin\theta - F_a - F_s}{\rho g}\right) + \left(A_1\bar{z}_1 + \frac{Q u_1}{g}\right) = \left(A_2\bar{z}_2 + \frac{Q u_2}{g}\right)$$

now we considering ~~small~~ small stretch of channel and considering smooth boundary, so $\theta \simeq 0$ and $F_s = 0$



and also $F_a \simeq 0$ (~~thus~~ negligible)

so the first term of left hand side of above eqn becomes zero,

$Q = A_1 u_1 = A_2 u_2$

$\Rightarrow u_1 = Q/A_1$, $u_2 = Q/A_2$ } substituting this in above eqn finally the above eqn becomes,

$$\left(A_1\bar{Z}_1 + \frac{Q^2}{A_1 g}\right) = \left(A_2\bar{Z}_2 + \frac{Q^2}{A_2 g}\right) \quad \text{——— (2)}$$

The unit of each ~~parame~~ part of above eqn is equal to Force per unit specific weight of fluid or water, $F/\gamma = \square = A_1\bar{Z}_1$, $Q^2/A_1 g$

Thus we can ~~say, that~~ denote above term as specific force, M

$$M_1 = M_2, \quad M_1 = A_1\bar{Z}_1 + \frac{Q^2}{A_1 g}$$

$$M_2 = A_2\bar{Z}_2 + Q^2/A_2 g$$

Thus sp. force at section ① - ① is equals to sp. force at section ② - ②
for a given discharge Q

$$\boxed{M_1 = M_2}$$

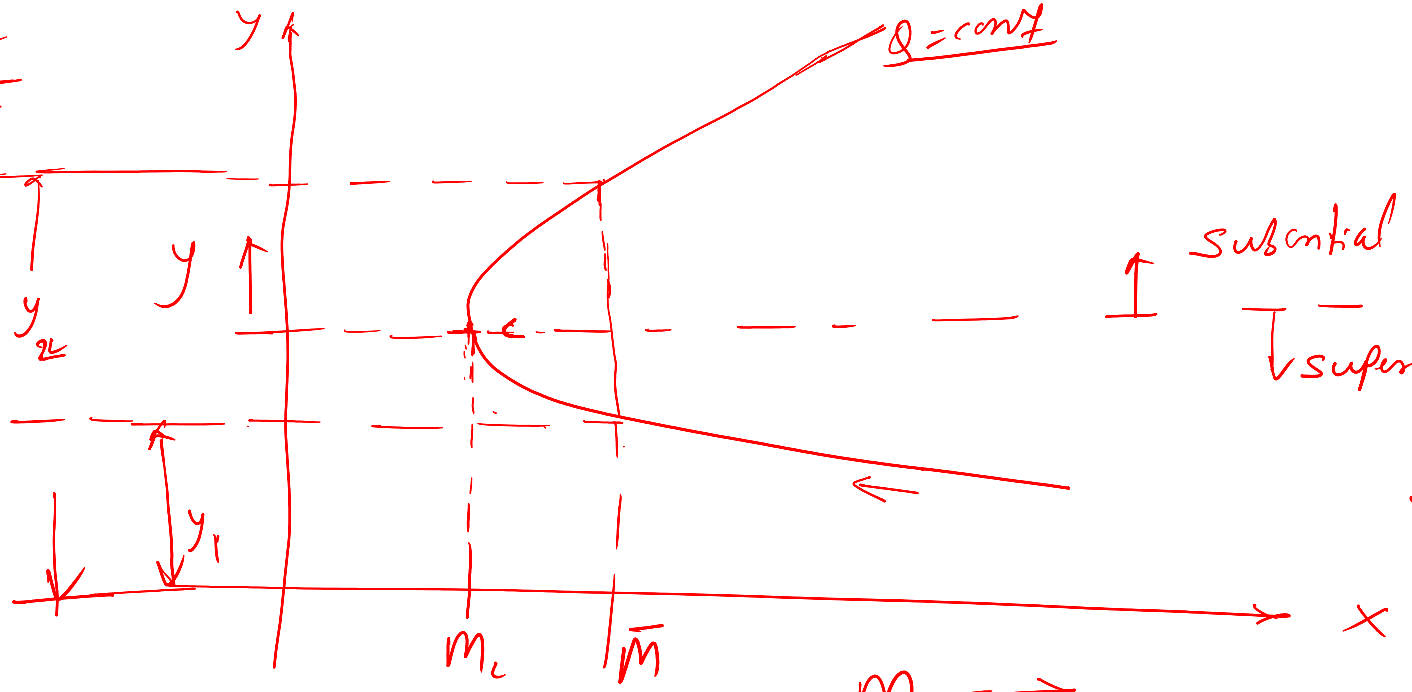
Specific Force diagram

$$M = A\bar{Z} + \frac{Q^2}{Ag}$$

for a given discharge (Q) and channel width B,
The left hand side of this eqn is function of depth of channel
i.e. $M = f(y)$ ~~or~~

↓

$$M = A\bar{z} + \frac{Q^2}{Ag}$$



y_1 & y_2 are known as conjugate depth or sequent depth and it occurs at $M > M_c$

~~As~~ Initially sp. force decreases with increase in flow depth y, and it reaches to a particular point at which sp. Force becomes minimum. ~~then after, wit~~ Then further increase in flow depth beyond this point C, sp. Force increases with increase in flow depth.

~~Now~~ now, we need to find the depth at which sp. force becomes minimum.

for this differentiating M, w.r.t y and equating with zero

$$\frac{dM}{dy} = 0$$

$$\Rightarrow \frac{dM}{dy} = \frac{d}{dy}\left(A\bar{Z} + \frac{Q^2}{Ag}\right)$$

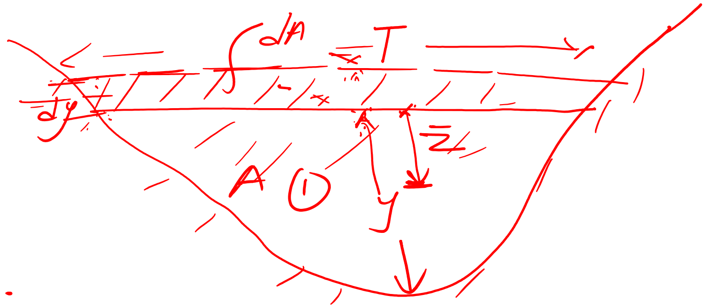
$$= \frac{d(A\bar{Z})}{dy} + \frac{d}{dy}\left(\frac{Q^2}{Ag}\right)$$

$$= \frac{d(A\bar{Z})}{dy} + -A^{-2}\frac{Q^2}{g}\frac{dA}{dy}$$

$$\frac{dM}{dy} = \frac{d(A\bar{Z})}{dy} - \frac{Q^2}{A^2}\frac{dA}{dy}$$

$$\frac{dM}{dy} = \frac{-Q^2}{gA^2}\frac{dA}{dy} + \frac{d(A\bar{z})}{dy}$$

$(A\bar{z})$



$A\bar{z}$ = First moment of Area.

$\frac{d(A\bar{z})}{dy_x}$ = change in first moment of Area w.r.t y

= Small increase in $(A\bar{z})$ with small increment of flow depth y

$$= \frac{(A(\bar{z}+dy) + (dA * \cdot dy/2)) - (A\bar{z})}{dy}$$

$$= \frac{A(\bar{z}+dy) + (Tdy \cdot dy/2) - (A\bar{z})}{dy}$$

the term $(dy)^2/2$ is small so neglected it)

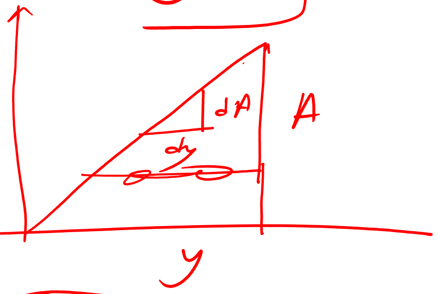
$$\frac{d(A\bar{z})}{dy} = \frac{A(\bar{z}+dy) + 0 - (A\bar{z})}{dy}$$

$$= \frac{A\,dy}{dy} = A$$

$$\boxed{\frac{A}{y} = T}$$

$$\Downarrow$$

$$\boxed{T = \frac{dA}{dy}}$$



$m = y/x$

$m = \frac{dy}{dx}$

$$T = A/y$$

$$T = \frac{dA}{dy}$$

$$\frac{dM}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + A$$

$$= -\frac{Q^2}{gA^2}\cdot T + A$$

for min sp. force

$$\frac{dM}{dy} = 0$$

$$+\frac{Q^2 T}{gA^2} = +A \Rightarrow \frac{Q^2 T}{gA^3} = 1$$

$\frac{Q^2 T}{g A^3} = 1$, It means that flow will be critical

and at critical flow, Sp. Force will be minimum for given discharge

$\frac{Q^2 T}{g A^3} = 1 \Rightarrow F_r^2 = 1 \Rightarrow F = 1$

$\frac{V}{\sqrt{g y_c}} = 1 \Rightarrow y_c = \boxed{}$

$$M = \frac{Q^2}{Ag} + A\bar{Z}$$ $\rightarrow$

$Q \Rightarrow$ ~~yc~~

$$\frac{Q^2}{gA^3} = 2$$ ~~yc~~

$$\Rightarrow \frac{Q^2}{Ag} = -(M - A\bar{Z})$$

$$\Rightarrow Q = \sqrt{Ag}\sqrt{(M - A\bar{Z})} \quad \text{——} (1)$$

now we need to know the condition for discharge to be maximum, for ~~the differentiate~~ for given specific force.

Differentiate Q w.r.t y, and equates its with zero,

$$\frac{dQ}{dy} = 0$$

$$\Rightarrow \frac{d(\sqrt{Ag}\sqrt{M - A\bar{Z}})}{dy} = 0$$

$$\frac{dQ}{dy} = \sqrt{Ag}\,\frac{d\left(\sqrt{(m-A\bar{z})}\right)}{dy} + \sqrt{m-A\bar{z}}\,\frac{d\sqrt{Ag}}{dy}$$

$$\frac{d(A\bar{z})}{dy} = A$$

$$= \sqrt{Ag} \times \frac{1}{2}(m-A\bar{z})^{(1/2-1)} \times \frac{d(m-A\bar{z})}{dy} + \left(\sqrt{m-A\bar{z}}\right)\sqrt{g}\,\frac{d A^{1/2}}{dy}$$

$$= \sqrt{Ag}\,\frac{1}{2\sqrt{m-A\bar{z}}} \times \left(0 - \frac{d(A\bar{z})}{dy}\right) + \sqrt{g}\sqrt{m-A\bar{z}} \times \frac{1}{2\sqrt{A}} \times \frac{dA}{dy}$$

$$\frac{dQ}{dy} = \frac{-\sqrt{Ag}}{2\sqrt{m-A\bar{z}}} \times A + \frac{\sqrt{g}\sqrt{m-A\bar{z}}}{2\sqrt{A}} \times T = 0$$

$$\frac{\cancel{\sqrt{g}}\,A\sqrt{A}}{\cancel{2}\sqrt{m-A\bar{z}}} = \frac{\cancel{\sqrt{g}}\,T\sqrt{m-A\bar{z}}}{\cancel{2}\sqrt{A}} \Rightarrow A^2 = T(m-A\bar{z})$$

$$\frac{A^2}{T} = m - A\bar{z}$$

$$\Rightarrow \frac{A^2}{T} = \frac{Q^2}{Ag}$$

$$\Rightarrow \frac{Q^2 T}{A^3 g} = 1 \Rightarrow \text{critical flow condn.}$$

Thus we can say that the channel will pass maximum discharge at critical flow condn. for given specific force.

* critical depth computation, At critical flow,

- Sp. Energy is minimum for given discharge
- Discharge is maxm. for given Sp. Energy
- Sp. Force is minimum for given discharge
- Discharge is maxm. for given Sp. Force
- Froude no. is unity

← At critical flow

$$\frac{Q^2 T}{g A^3} = 1 \longrightarrow F_r = 1 \Rightarrow \frac{V}{\sqrt{g y_c}} = 1$$

$$V^2 = g y_c \Rightarrow$$

$$(B y_c = A_c)$$

$$\frac{Q^2}{A^2} = g y_c$$

$$\frac{Q^2}{B^2 y_c^2} = g y_c \Rightarrow y_c = \square$$

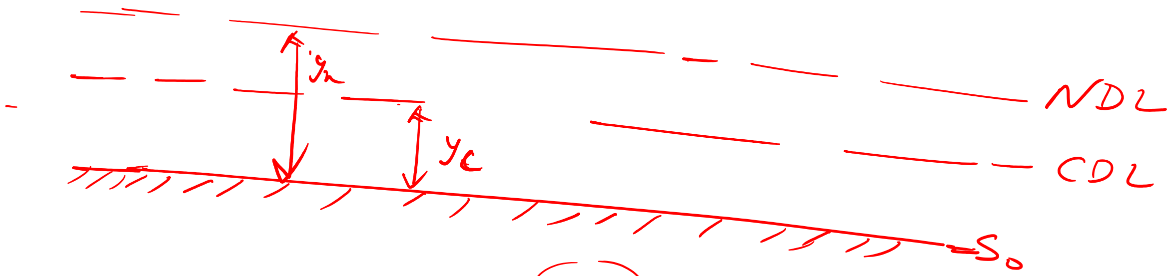
$$R = \frac{B y_c}{B + 2 y_c}$$

Critical slope $\rightarrow$

$$V = \frac{1}{n} R^{2/3} S_c^{1/2}$$

$$\Rightarrow S_c = \square$$

- Mild slope
- Steep slope
- critical slope



$S_o < S_c < S_s$

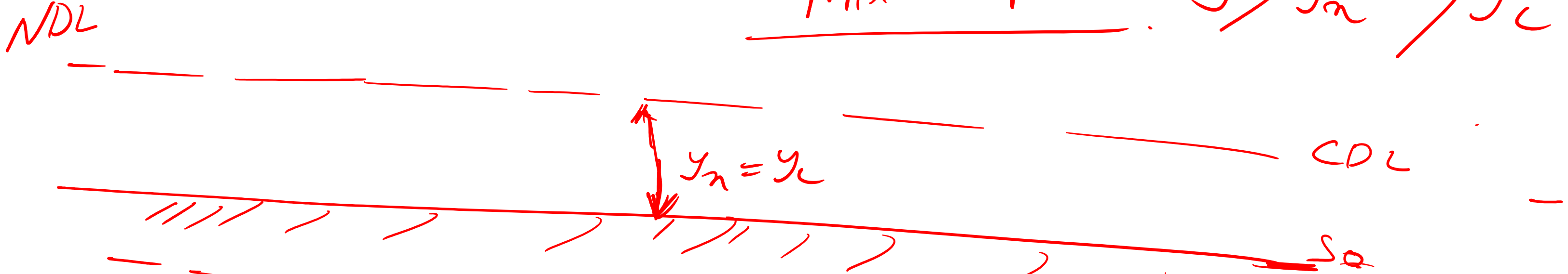
$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$\Rightarrow y_n =$ normal

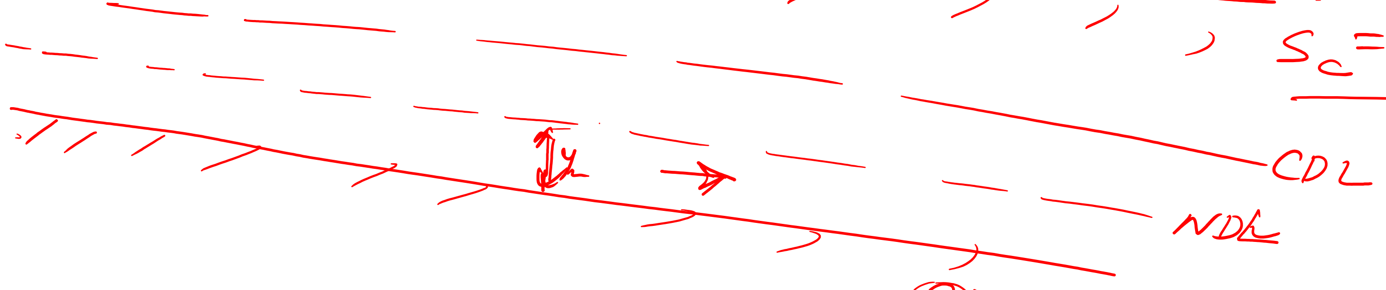
$y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$S_o < S_c < S_s$

Mild slope $\quad y > y_n > y_c$



$S_c = S_o$



$S_s > S_c > S_m$