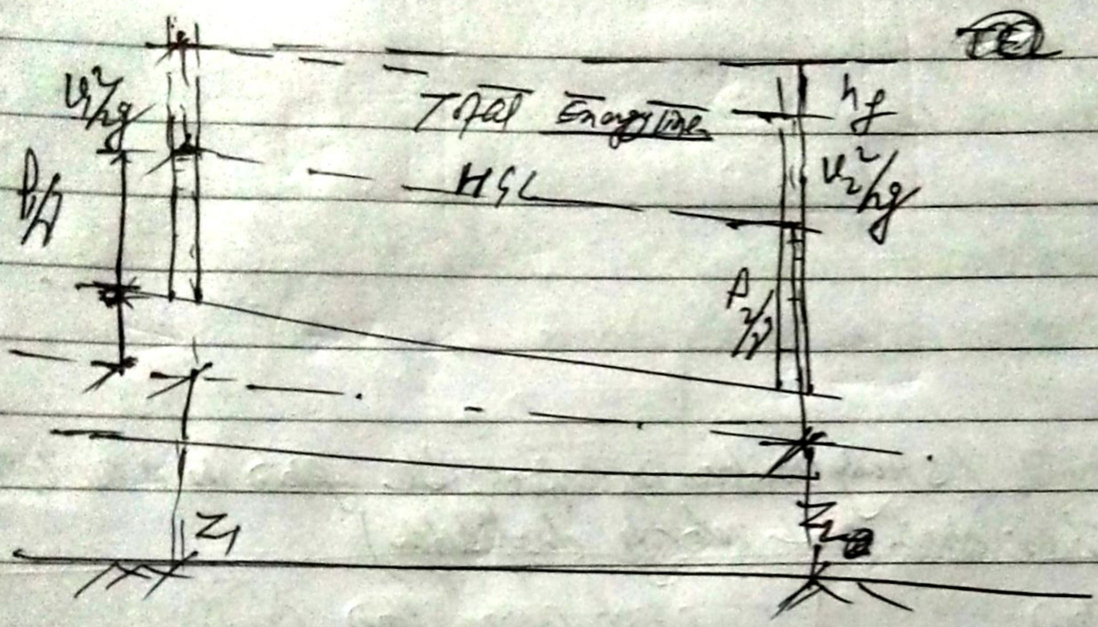
Chapter - 1

Introduction

Types of flow

Pipe flow

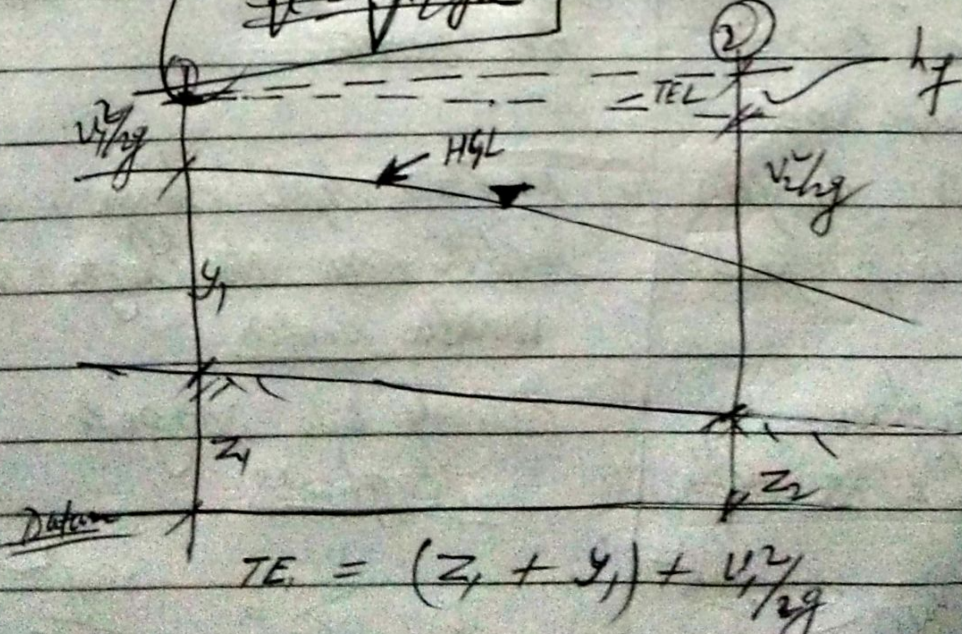


$\boxed{\text{Pitot tube}}$ to measure velocity head $TE = \left(Z_1 + P_1/\gamma\right) + V_1^2/2g$

open channel flow

$$\cancel{P = \rho g h} = \frac{1}{2}\rho V^2$$

$$\boxed{\cancel{V = \sqrt{2gh}}}$$



$$TE = \left(Z_1 + y_1\right) + V_1^2/2g$$

Hydraulics

pipe flow

1. Introduction to pipe flow
   - pipe and open channel flow.
   - Reynolds experiment and flow based on Reynolds number.

2. Laminar flow (steady uniform incompressible flow in a circular pipe (shear stress and velocity distribution)

Pipe flow	Open channel flow
(1) No free surface, pipe always full, has gauge pressure.	(1) presence of free surface, exposed to atmospheric pressure.
(2) Flow due to energy grade	(2) Flow under gravity
(3) Hydraulic grade line, ie. piezometric head $\left(\frac{p}{\gamma}+z\right)$ line, can be above or below pipe axis.	(3) HGL coincides with the free surface.
(4) for a given pipe, the flow takes place in a prefixed x-section.	(4) as flow depth changes, flow section area also changes, ie. variable section
(5) the boundary roughness is fairly low with in reasonable order. generally $k_s/\delta' \le 0.25$ in pipe flow	extremely wide range of roughness. $k_s/\delta' \ge 6$ rough

$\tau = \mu \frac{du}{dy}$

$\Rightarrow \frac{F}{A} = \mu \frac{du}{dy}$

$\Rightarrow F = \mu \frac{du}{dy} * A$

$0.25 \leq k_s/\delta' \leq 6$

$\frac{k_s}{\delta'} \geq 6$

(3) flow is governed by Reynold's number since flow is almost laminar due to influence of fluid viscosity	Froude number. ↑ prime nondimensional number governing the flow

Reynold's number $= \frac{\text{Inertial force}}{\text{viscous force}}$

$= \frac{m*a}{\mu \frac{du}{dy} * A}$

$= \frac{m * \frac{du}{dt}}{\mu \frac{du}{dy} * A}$

$= \frac{\cancel{A} * dl * \rho \frac{du}{dt}}{\mu \frac{du}{dy} * \cancel{A}} =$

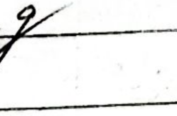
$= \frac{\rho v\, dy}{\mu}$ → depth

$= \frac{\rho v D}{\mu}$

Froude number = Ratio of square root of inertial force to gravity force

$$= \sqrt{\frac{F_i}{F_g}}$$

$$= \sqrt{\frac{m*a}{m*g}} \quad \cancel{= \sqrt{\frac{v}{t} * g}}$$



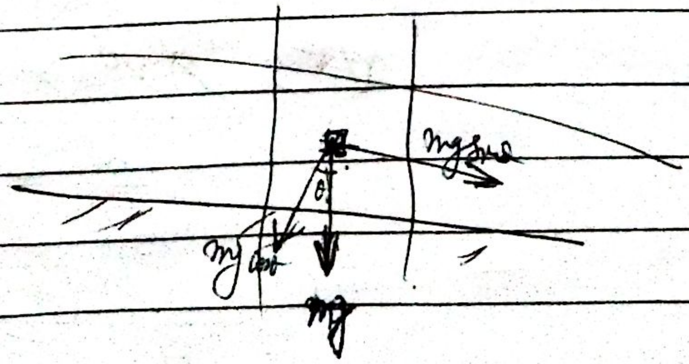
$F_i = m*a$

$$= \rho * V * \frac{v}{t} = \rho * A * \frac{l}{t} * v$$

$$= \rho A v^2$$

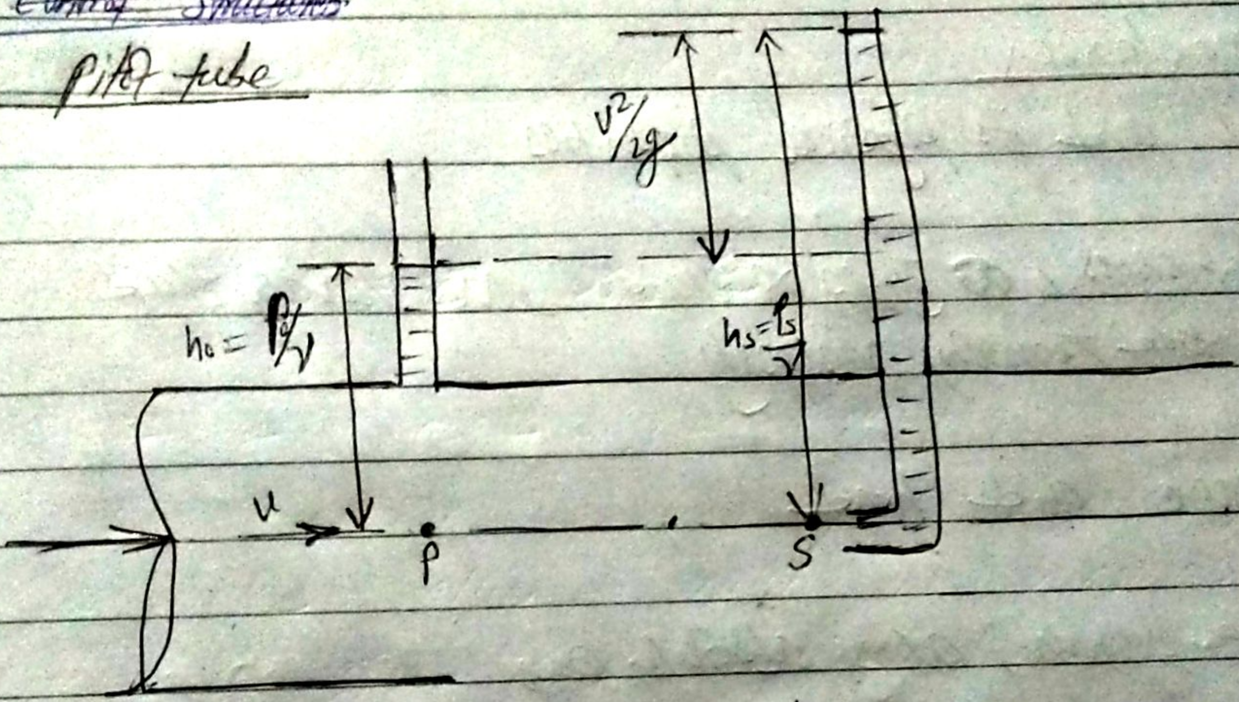
$F_g = m*g = \rho * V * g = \rho * A * l * g$

$$F = \sqrt{\frac{\rho A v^2}{\rho A l g}} = \frac{v}{\sqrt{g d}}$$



~~Erosive and hydrodynamic processes downstream of a low head control structures~~

Pitot tube



P_s = stagnation pressure

P_0 = static pressure.

applying BE at P & S

$$z + \frac{P_0}{\gamma} + \frac{v^2}{2g} = z + P_s/\gamma$$

$$\therefore \frac{v^2}{2g} = \left(\frac{P_s - P_0}{\gamma}\right) = \Delta h$$

$$\boxed{v^2 = 2g\,\Delta h}$$

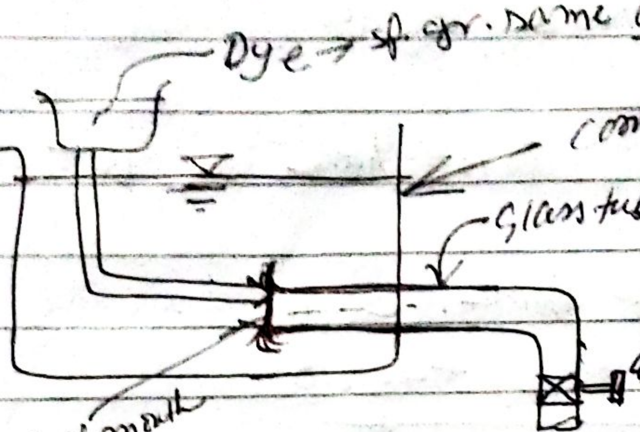
$$\therefore v = \sqrt{2g\Delta h}$$

Δh = Dynamic pressure

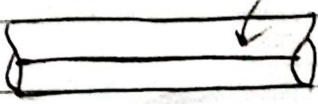
Reynolds experiment

Osborne Reynolds in 1883

to find out two types of flow with the help of dye.

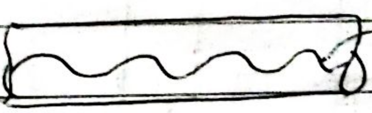


Dye filament



1/ at low flow velocity dye remain straight and stable (a) Laminar flow → at low velocity $v < V_{cr}$

(b)



wavy filament

2/ $v >$ increasing velocity, a critical state — Transitional flow $v \approx V_{cr}$

3/ (c)

$v >> V_{cr}$, The fluctuation in the filament of dye become more intense and ultimately the dye diffused over the entire x-section of the tube, due to intermingling of the particles

Turbulent flow — Diffused filament

observation made

~~on~~ on the basis of his expriment, Reynolds discover that

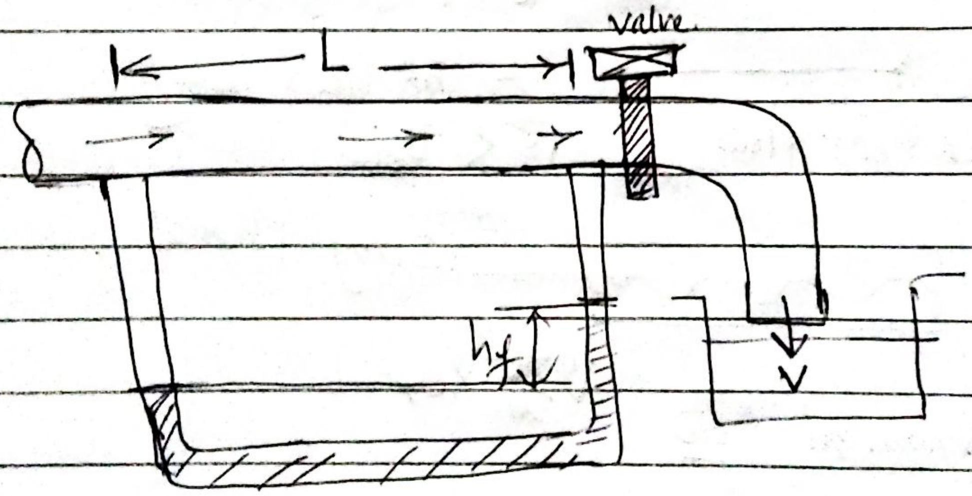
<u>at laminar flow</u>

(i) The loss of pressure head $\propto v$

(ii) at turbulent flow loss of head $\propto v^2$

more exactly loss of head $\propto v^n$

$n \approx 1.75$ to 2.0



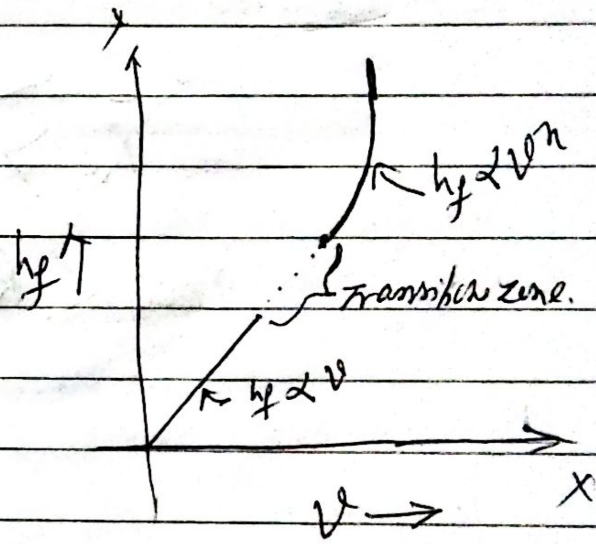
Apparatus used by Reynolds for estimating the loss of head in a pipe by measuring the pressure difference over a known length of pipe

$$Q = \frac{\text{volume poured in (tank)}}{\text{time (stop watch)}}$$

now velocity of flow $v = Q/A$ (x-section Area of pipe)

increase or decrease the rate of flow by regulating valve and find out different set of velocity and note corresponding head loss

v	h_f
v_1	h_{f1}
v_2	h_{f2}
v_3	⋮
↓	↓



At lower velocity $h_f \propto v$ (laminar)

At higher velocity $h_f \propto v^n$ $n = 1.75$ to 2.0

flow is turbulent

Reynolds number

Reynold's from his experiment found that the nature of flow in a closed conduit depends on (i) Diameter of pipe (D)

ρ (fluid)

(ii) viscosity

v (velocity)

By combining above variables, Reynolds determined non dimensional number $R = \frac{\rho v D}{\mu}$

in general case D is replaced by L known as characteristic length

$$Re = \frac{\rho V L}{\mu}$$

$(\mu/\rho) \rightarrow$ kinematic viscosity

$Re < 2000 \Rightarrow$ Laminar

$Re > 4000 =$ Turbulent

$2000 < Re < 4000 \Rightarrow$ unpredictable

Transition flow

2. Laminar flow (steady uniform incompressible flow in a circular pipe, shear stress and velocity distribution)

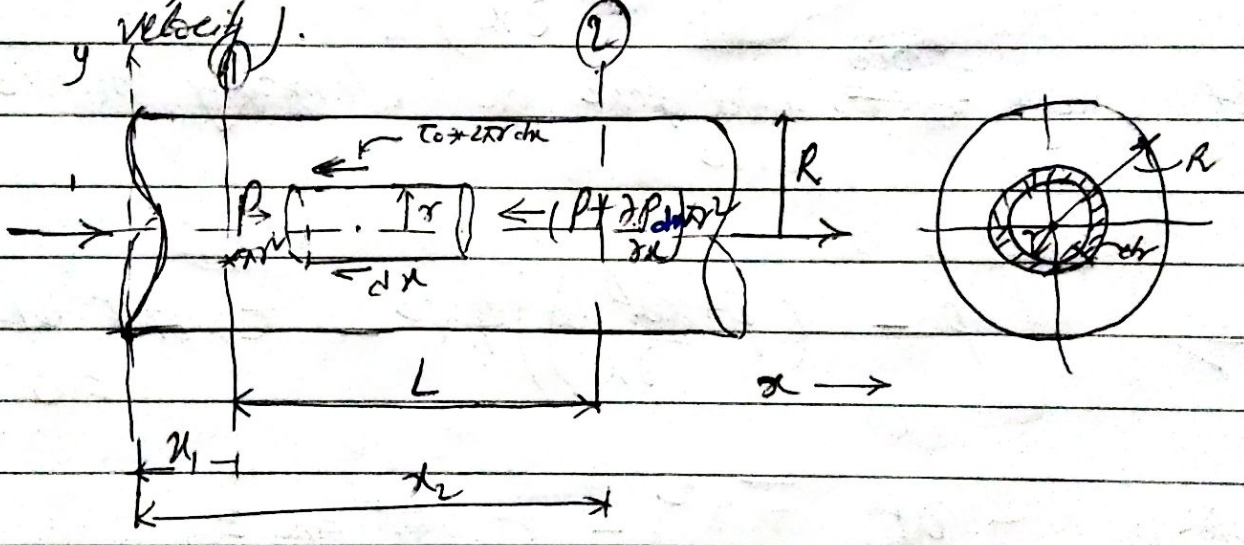
3. Head loss, Hagen poisseuille equation

* Hagen-poiseuille theory is based on the following assumption:

1. The fluid follows Newton's Law of Viscosity

i.e. $\tau = \mu \frac{du}{dy}$

2. There is no slip of fluid particles at the boundary (i.e. the fluid particles adjacent to the pipe will have zero velocity).



~~Forces acting~~ Let us consider a pipe of Radius R through which fluid is flowing having laminar flow. considering elementary ~~ele~~ concentric cylinder of radius r (fluid element) and length dx as free body.

Now the forces acting on ~~Left side end~~ fluid element

(i) shear force
(ii) pressure force

Shear force at boundary of current cylinder
$= \tau * 2\pi r \, dx$

(ii) pressure force at left end
$= P \times \pi r^2$

(iii) Pressure force at right end
$= \left(P + \frac{\partial P}{\partial x} dx\right) * \pi r^2$

at steady ~~state~~ fluid, net force on elementary must be zero

$\Sigma F_x = 0$

$$\Rightarrow \left(P - \left(P + \frac{\partial P}{\partial x} dx\right)\right) \pi r^2 - \tau \times 2\pi r \, dx = 0$$

or, $-\frac{\partial P}{\partial x} \pi r^2 dx - \tau \, 2\pi r \, dx = 0$

$$\Rightarrow \pi r \, dx\left(-\frac{\partial P}{\partial x} r - 2\tau\right) = 0$$

$\pi r \, dx \neq 0$

$$-2\tau - \frac{\partial p}{\partial x} r = 0$$

$$\Rightarrow \tau = -\frac{\partial p}{\partial x} \frac{r}{2}$$

at $r = 0 \Rightarrow \tau = 0$

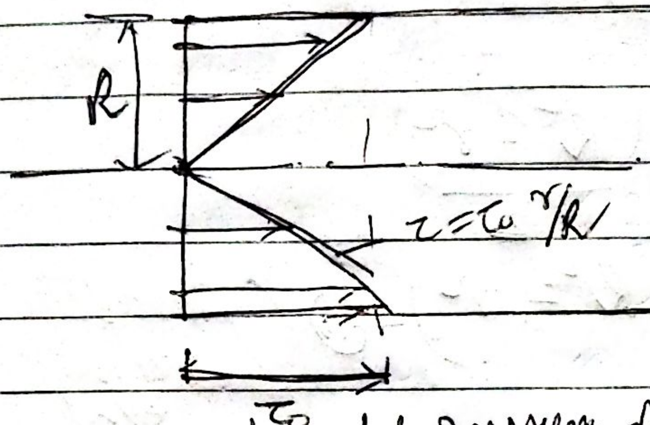
at $r = R$ $\quad \tau_0 = -\frac{\partial p}{\partial x} \frac{R}{2}$ — (I)

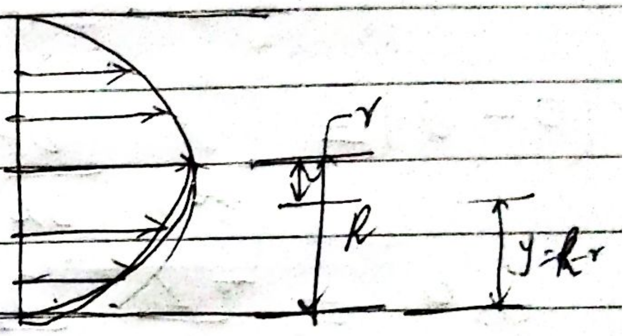
$\tau = \frac{\partial p}{\partial x} \frac{r}{2}$

$\tau_0 = \frac{\partial p}{\partial x} \frac{R}{2}$

$\frac{\tau}{\tau_0} = r/R$

$\Rightarrow \tau = \tau_0 \, r/R$





– ve sign shows that pressure decreases as distance increases (decrease in the dirn. of flow)

Shear stress distribution

velocity distribution

$$\tau_0 = -\frac{\partial p}{\partial x}\left(\frac{R}{2}\right) \quad \text{— (II)}$$

from Newton law of viscosity

$$\tau = \mu \frac{du}{dy} \quad \text{— (III)}$$

In this eqn distance y is measured from boundary

so $y = R - r$

$dy = -dr$

$\Rightarrow$ (II) becomes

$$\tau = -\mu \frac{du}{dr}$$

$$+\mu \frac{du}{dr} = +\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$du = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{r}{2}\,dr$$, integrating from u to 0

velocity at r = u
and at R = 0 (boundary)

$$\int_u^0 du = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\int_r^R r\,dr$$

$$(0 - u) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{(R^2 - r^2)}{2}$$

$$u = \frac{-1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \text{ --- (IV)}$$ (parabolic curve eqn)

$\Rightarrow$ $\cancel{\left[u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\right]}$ —

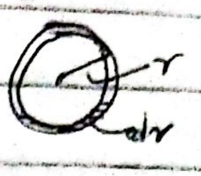
u_{max} at $r = 0$

so $$u_{max} = \frac{-1}{4\mu}\frac{\partial p}{\partial x}R^2 \text{ --- (V)}$$

$$\frac{u}{u_{max}} = \frac{R^2 - r^2}{R^2}$$

$$\boxed{u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]}$$

now discharge through elementary ring of thickness dr at radial distance r is given by



$$dQ = u \times 2\pi r \cdot dr \quad — \text{(VI)}$$

from eqn (V) & (VI)

$$dQ = U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] 2\pi r \, dr$$

Total discharge, by Integration

$$\int dQ = 2\pi U_{max} \int \left(1-\left(\frac{r}{R}\right)^2\right) r \, dr$$

$$\int dQ = 2\pi U_{max} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$Q = 2\pi U_{max}\left[\frac{R^2}{2} - \frac{1}{R^2}\left[\frac{R^4}{4} - 0\right]\right]$$

$$= 2\pi U_{max}\left(\frac{R^2}{2} - \frac{R^2}{4}\right)$$

$$Q = \cancel{2}\pi U_{max} \frac{R^2}{\cancel{4}\,2} = \frac{\pi}{2} U_{max} R^2$$

Avg. velo. of flow $\bar{u} = Q/A = \dfrac{\frac{\pi}{2} U_{max} R^2}{\pi R^2}$

$$\bar{u} = \frac{U_{max}}{2} \quad — \text{(VIII)}$$

from eqn (V) & (VII)

$$2\bar{u} = \frac{-1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$\Rightarrow \cancel{\frac{\partial p}{} = \frac{1}{8\mu}}$$

$$\partial p = \frac{-8\mu\bar{u}\,\partial x}{R^2}$$

$$-\int_{p_1}^{p_2} \partial p = \frac{8\mu\bar{u}}{R^2} \int_{x_1}^{x_2} \partial x$$

$$-(p_2 - p_1) = \frac{8\mu\bar{u}}{R^2}(x_2 - x_1)$$

$$(p_1 - p_2) = \frac{8\mu\bar{u}\,L}{(D/2)^2}$$

$$\boxed{(p_1 - p_2) = \frac{32\mu\bar{u}\,L}{D^2}}$$

<u>Hagen Poiseuille eqn</u>

$$\left(z_1 + \frac{p_1}{\gamma}\right) - \left(z_2 + \frac{p_2}{\gamma}\right) = h_f$$

$$z_1 = z_2 \Rightarrow \frac{(p_1 - p_2)}{\gamma} = h_f \Rightarrow p_1 - p_2 = \gamma h_f$$

Head loss

now using B.E. eqn

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_f$$

now for pipe of uniform x-section

$$V_1 = V_2$$

and at same elevation $Z_1 = Z_2$

$$\Rightarrow \frac{P_1 - P_2}{\gamma} = h_f$$

$$P_1 - P_2 = \gamma h_f$$ substitute this in Hagen Poiseuille eqn

we get

$$h_f = \frac{32 \mu \bar{u} L}{\gamma D^2}$$

where γ = sp. weight of ~~water~~ fluid

$$\gamma = 9.81 \text{ kN/m}^2$$

$$\gamma = \rho g$$

$$= (kg/m^3) \times m/s^2$$

$$= kg/m^2 s^2$$