Hydraulics

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CONTENTS

- ✓ Definition, difference between pipe flow and open channel flow
- ✓ Reynold’s experiment, Reynold’s number, Hydraulic Radius
- ✓ Laminar flow
- ✓ Hagen- Poiseuille equation and Hagen- William equation
- ✓ Turbulent flow,Shear stress development, Reynolds stress
- ✓ Prandtl mixing length theory, Velocity distribuition in turbulent flow
- ✓ Darcy –Weisbach equation, hydrodynamically smooth and rough boundary
- ✓ Nikuradse experiments and results discussion
- ✓ Coolebrook-White equation, Swami – Jain equation and Moody’s chart,
- ✓ Minor head losses and pipe flow problem.

CONTENTS

❑ Pipe flow

- A pipe is a closed conduit , used for carrying fluids under pressure.
- It always runs full.

❑ Open Channel Flow

Open channel is a conduit in which liquid flows with a free a free surface under gravity.

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Fig.1: It shows open channel flow as well as pipe flow (Syphon) for irrigation

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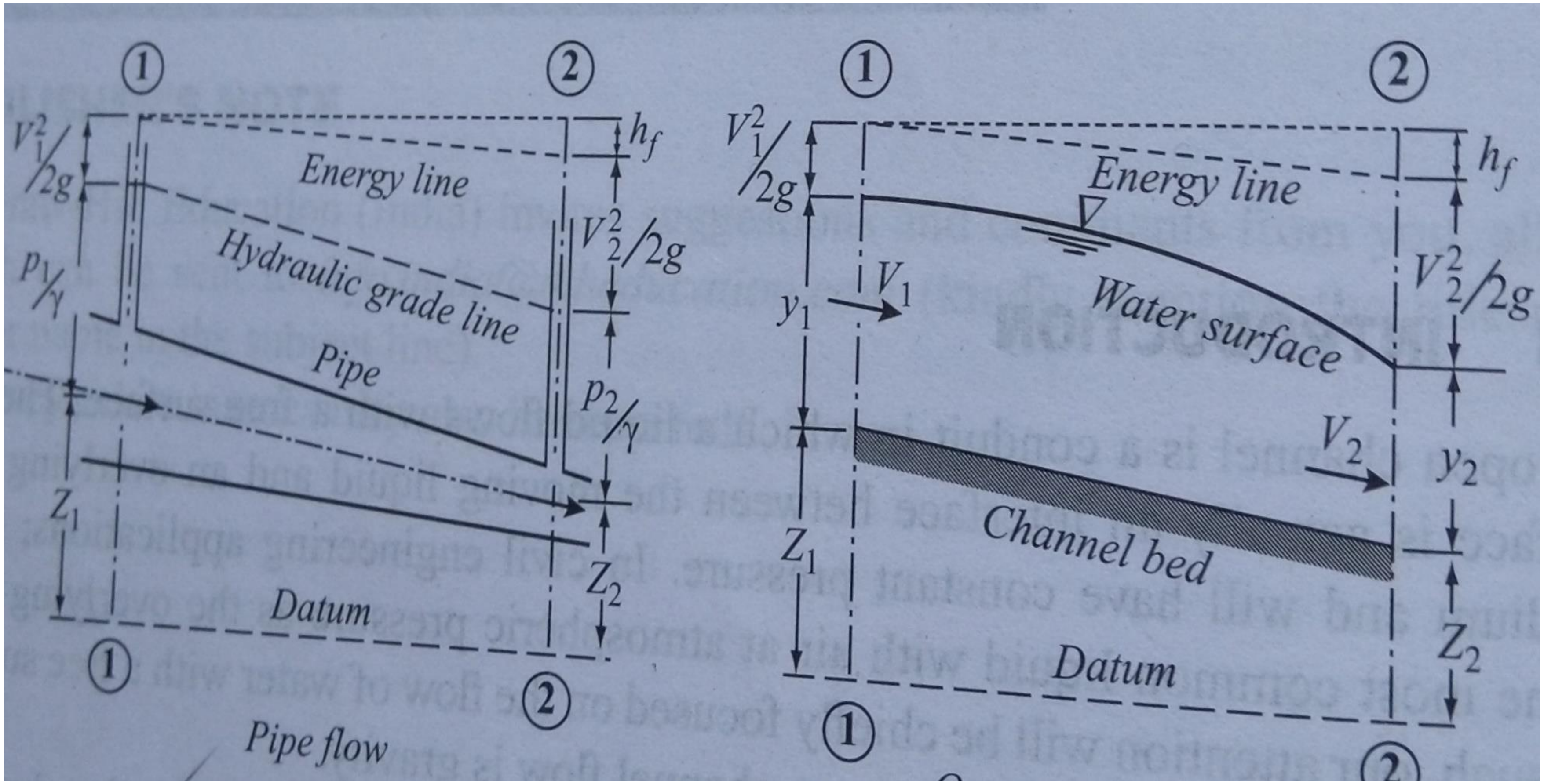

Fig. 2: Schematic representation of Pipe flow and Free surface flow(open channel)

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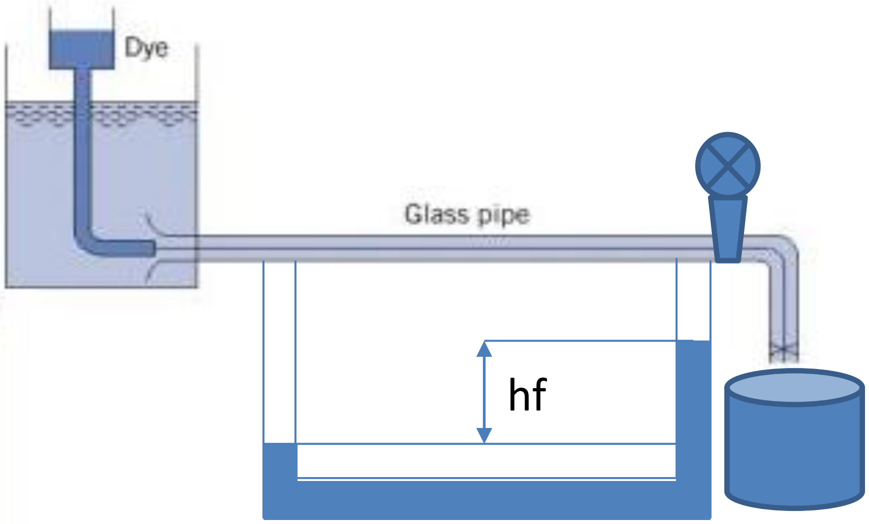
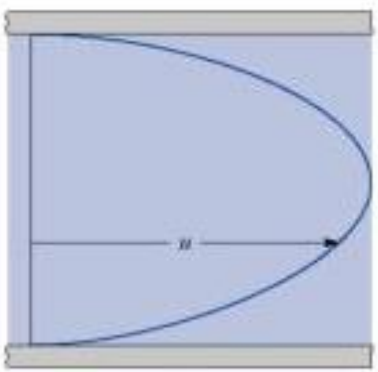
Reynolds Experiment

- Reynolds Number
- Laminar flow: Fluid moves in smooth streamlines
- Turbulent flow: Violent mixing, fluid velocity at a point varies randomly with time
- Transition to turbulence in a 2 in. pipe is at *V*=2 ft/s, so most pipe flows are turbulent

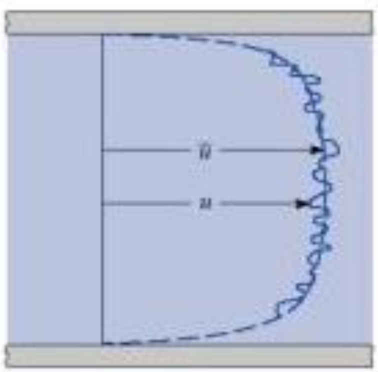
$$
Re = \frac{\rho V D}{\mu} \begin{cases} < 2000 & \text{Laminar flow} \quad h_f \propto V \\ 2000-4000 & \text{Transition flow} \\ > 4000 & \text{Turbulent flow} \quad h_f \propto V^2 \end{cases}
$$





(a)

Laminar



(b)

Turbulent

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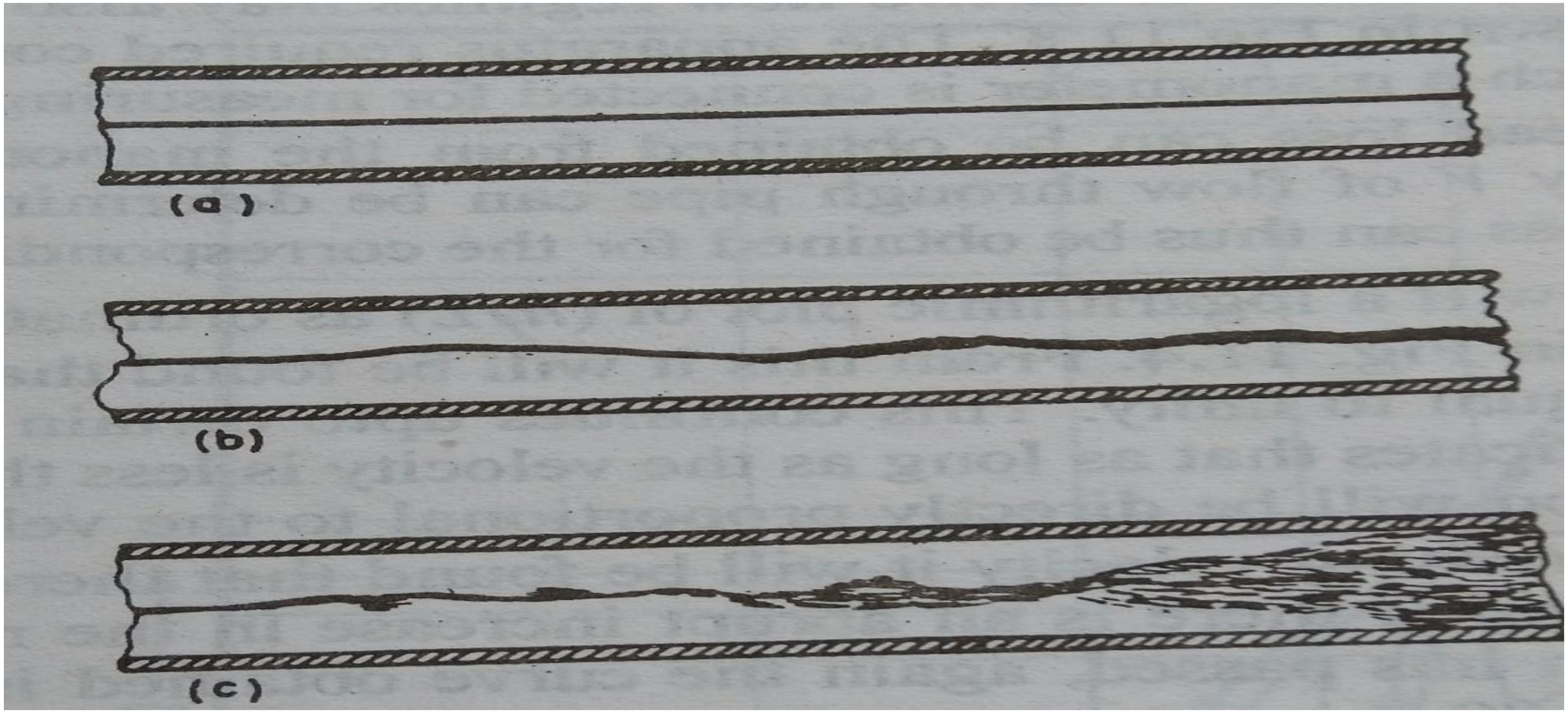

Fig. 3: Straight filament of dye represents laminar flow in (a), wavy filament in (b) shows transition nature of flow and fig (c) depicts the dispersing of wavy filament with water layer(turbulent)

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Q= Volume of water collected/Time measured using stop watch

Velocity, v = Q/A ; A= X sectional area of pipe

hf = head loss measured using manometer for several values of v

Sn.	Time (t) (s)	Volume (m3)	Discharge (Q)	Velocity, v	head loss (m)
1					
2					
3					
4					
5					
6					

Contd:

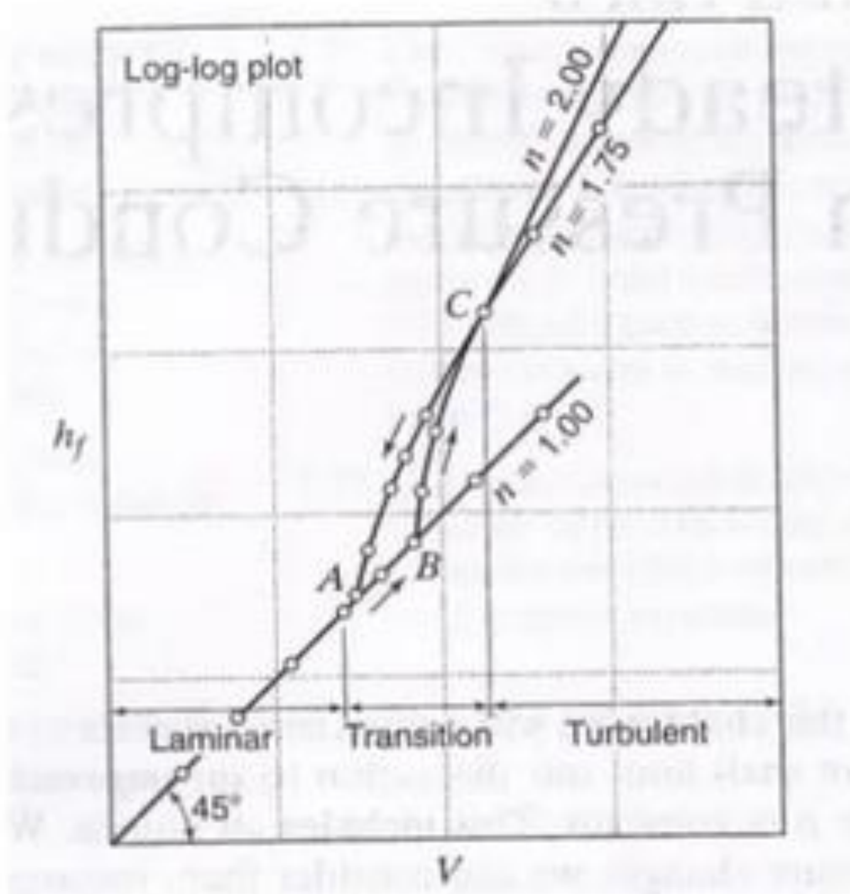
The head loss due to friction in a given length of pipe is proportional to mean velocity of flow (V) as long as the flow in laminar. i.e.,

$$H_f \propto V$$

But with increasing velocity, as the flow become turbulent the head loss also varies and become proportion to V^n

$$H_f \propto V^n$$

Where n ranges from 1.75 to 2



Log-log plot for flow in uniform pipe (n=2.0 for rough wall pipe; n=1.75 for smooth wall pipe

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- **Reynold's Number(*R or Re*):** It is ratio of inertial forces (Fi) to viscous forces (Fv) of flowing fluid

$$Re = \frac{Fi}{Fv} = \frac{Mass.\frac{Velocity}{Time}}{Shear\ Stress.\ Area} = \frac{\rho \frac{Volume}{Time}.Velocity}{Shear\ Stress.\ Area}$$

$$= \frac{\rho Q.V}{\tau.A} = \frac{\rho AV.V}{\mu \frac{du}{dy}.A} = \frac{\rho AV.V}{\mu \frac{V}{L}.A} = \frac{\rho VL}{\mu} = \frac{VL}{\upsilon}$$

$$R_e = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

Where ;
V is avg. velocity of flow in pipe
ν is kinematic viscosity
L is characteristic/representative linear dimension of pipe. It is diameter of pipe (circular conduits) or hydraulic radius (non-circular conduits).

- For laminar flow: Re<=2000
- For transitional flow: 2000<Re<4000
- For Turbulent flow: Re>= 4000

Values of critical Reynolds no.

Note: For non-circular section, we need to use hydraulic radius (R_h) instead of diameter (D) for the linear dimension (L).

Contd:

- **Hydraulic Radius (R_h) or Hydraulic Diameter:** It is the ratio of area of flow to wetted perimeter of a channel or pipe

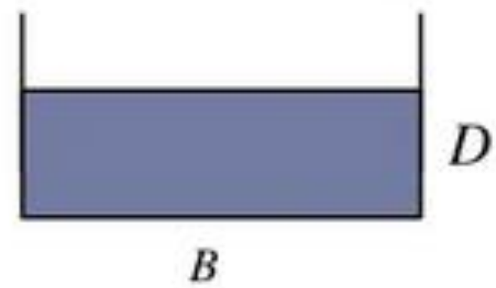
$$R_h = \frac{Area}{wetted\ perimeter} = \frac{A}{P}$$

For Circular Pipe

$$R_h = \frac{A}{P} = \frac{((\pi/4)D^2)}{\pi D} = \frac{D}{4}$$

$$D = 4R_h$$

For Rectangular pipe



$$R_h = \frac{A}{P} = \frac{BD}{B+2D}$$

$$R_h = \frac{VD}{\nu} = \frac{4VR_h}{\nu}$$

By replacing D with R_h, Reynolds' number formulae can be used for non-circular sections as well.

Note: hydraulic Radius gives us indication for most economical section. More the Rh more economical will be the section.

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- **Laminar flow (Steady uniform incompressible flow in a circular pipe, shear stress and velocity distribution)**
- **Head loss, Hagen Poiseuille equation & Hagen-William equation**

Hagen-Poiseuille theory is based on following assumption

1. Fluid follows Newton s law of viscosity .i.e., $\tau = \mu \frac{du}{dy}$ ....(1)
2. There is no slip of fluid particles at the boundary /wall .i.e., u=0

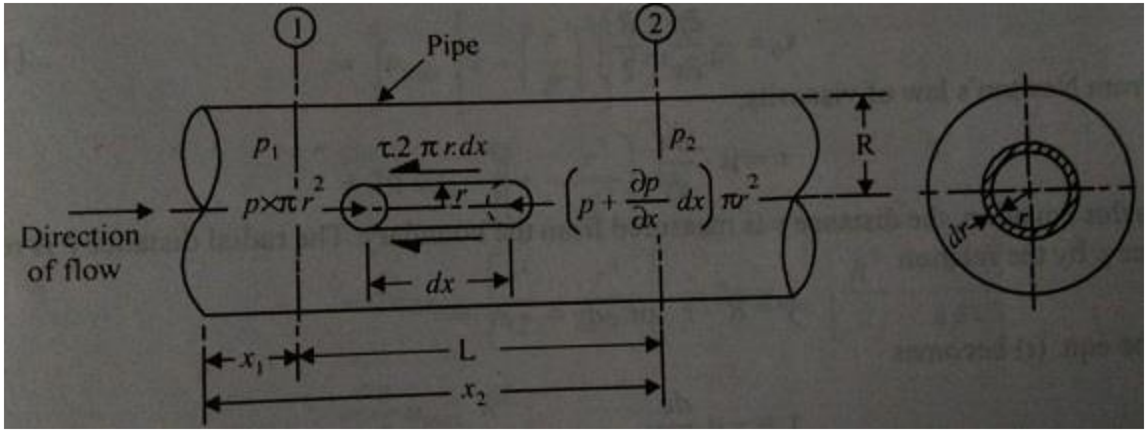

Fig. 4 : Laminar flow through elementary circular pipe

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Major forces acting on elementary pipe are:
Shear force at wall and Pressure force at two sections.

Considering all the forces, at steady flow net force on control volume must be zero.

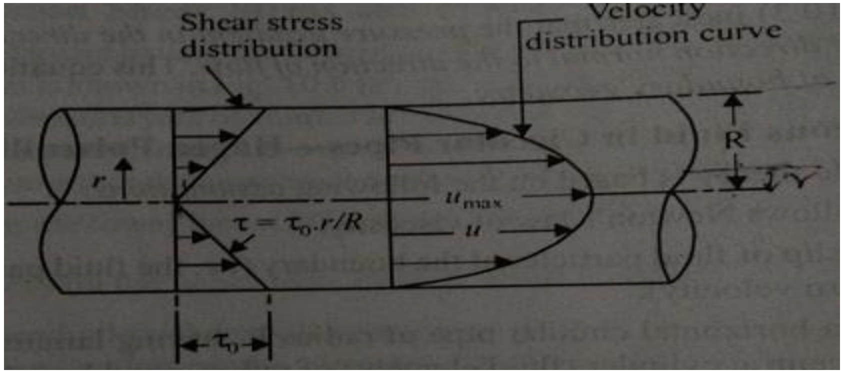
$$\sum Fx = 0$$

$$\left\{P - \left(P + \frac{\partial P}{\partial x} dx\right)\right\} \pi r^2 - \tau * 2 \pi r \, dx = 0$$

Simplifying it we get , $\tau = -\frac{dP}{dx} \times \frac{r}{2}$ .... (2)

at r=0, $\tau = 0$

at r=R, $\tau_o = -\frac{dP}{dx} \times \frac{R}{2}$



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Combining equation (1) and (2)and changing y in terms of r and rearranging we get,

$du = \frac{1}{\mu}\frac{\partial P}{\partial x}\frac{r}{2}dr$ ……………………………………(3)

Now integrating equation (3) from 0 to u and r to R at left and right hand side respectively. We get,

$u = -\frac{1}{4\mu}\frac{\partial P}{\partial x}(R^2 - r^2)$ ………………………………(4)

At center, r=0, u= u_{max}

Thus $u_{max} = -\frac{1}{4\mu}\frac{\partial P}{\partial x}(R^2)$ ……………………………(5)

Dividing (4)/(5),

$u = u_{max}\ (1-(r/R)^2)$ ……………………………………(6)

Further, to compute net discharge by integration, we get

$Q = \frac{\pi}{2} u_{max} R^2$ ………………………………………(7)

Contd:

$U_{avg} = \frac{umax}{2}$ … … … … … … … … … … … … …. (8)

From equation (5) and (8) and rearranging equation for computation of pressure difference for integration, we get

$-dP = \frac{8\mu ü}{R2}$ dx , integrating from P1 to P2 and length x1 to x2 and subtuting x2-x1 = l, we get

$(P_1 - P_2) = \frac{32\mu ü l}{D^2}$ …………………………………….(9)

Hagen- William equation

$V = kCR^{0.63}S^{0.54}$ ……………………… (10)

used to calculate velocity of fluid through pipes

Contd:

Where,

K= conversion factor for unit system

= 0.849 for m/s

= 1.318 for ft/s

C= factor for relative roughness, R= hydraulic radius

S= slope of the energy line

Turbulent Flow

- Turbulent flow, shear stress development, Reynolds stress
- Prandtl mixing length theory, velocity distribution
- Darcy weisbach equation
- Nikuradse experiment
- Moody's chart, Cole brook- White equation, Swami –Jain equation

Turbulence : It results from instability of laminar flow. It is random in nature where all quantities vary with time and space coordinates.

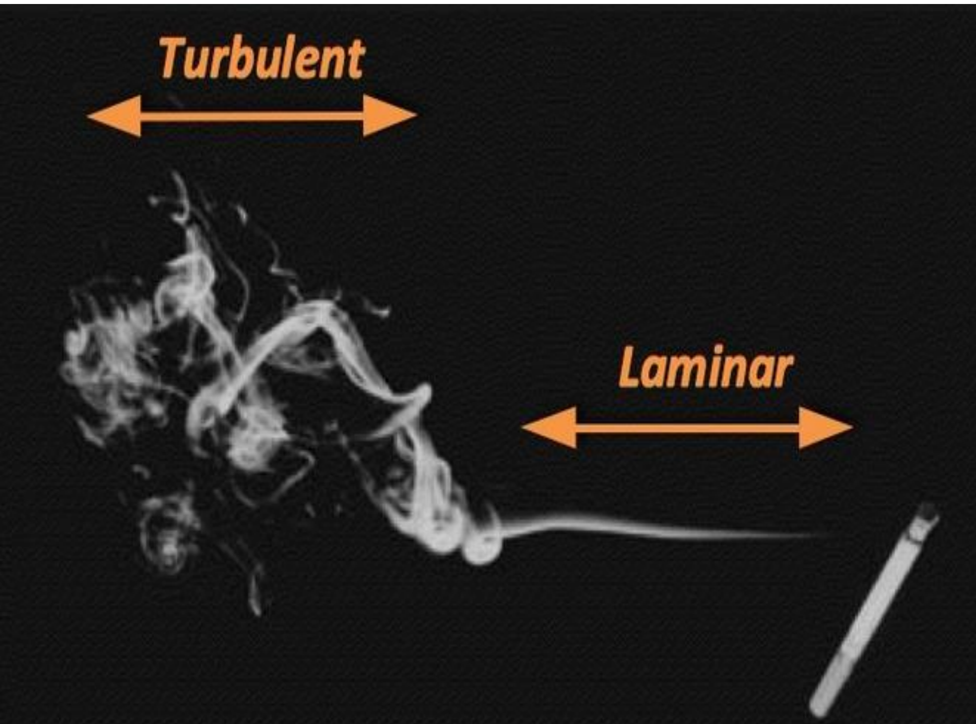

Fig. 6 : (a) Turbulence from mixing of smokes (b) Turbulence in a flume downstream of sluicegate and surface waves due to instability of flow , (hydraulics lab , IIT Roorkee). (c) Turbulence from breaking surface waves in Sunkoshi river along BP Highway

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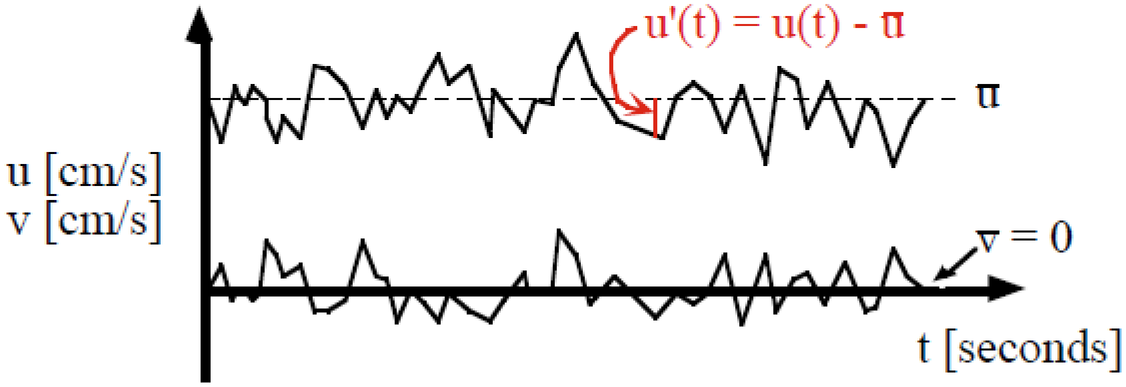

Fig. 7 : Velocity variation in quasi steady state

$$u(t) = \bar{u} + u'(t)$$

$$v(t) = \bar{v} + v'(t)$$

mean + turbulent fluctuation

Instantaneous velocity of flow along x, y and z direction

Contd:

Time average velocity at a point in a fluid flow is given as,

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt$$

Mean velocity:

$$\bar{u} = \int_t^{t+T} u(t)\,dt \qquad = \frac{1}{N}\sum_1^N u_i$$

continuous record   discrete, equi-spaced pts.

Turbulent Fluctuation:

$$u'(t) = u(t) - \bar{u} \quad \text{: continuous record}$$

$$u_i' = u_i - \bar{u} \quad \text{: discrete points}$$

Turbulence Strength:

$$u_{rms} = \sqrt{\overline{u'(t)^2}} = \sqrt{\frac{1}{N}\sum_{i=1}^N (u_i')^2}$$

Contd:

Shear stress due to turbulence:

Viscous shear stress $\tau = \mu \frac{du}{dy}$

Additional shear developed due to turbulence resulting from momentum transport is given by Boussinesq formula,

$\tau = \eta \frac{du}{dy}$, where η is coefficient of eddies viscosity

Total shear stress $\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy}$

Contd:

Reynold's stress

$\bar{u}_1 - \bar{u}_2 = u_a$ = relative velocity of layer A along x direction

Mass transfer per second from B to A ,

$M = \rho a v_a$

This mass attached on layer A and moves with relative velocity u_a along x direction

Thus momentum transfer along x direction = $\rho a v_a u_a$ ....(1)

Shear stress = $\rho a v_a u_a / a$,

$= \rho v_a u_a$

$\tau = -\rho v_a u_a$ ....(2) (-ve sign signifies that shear acts opposite to flow direction)

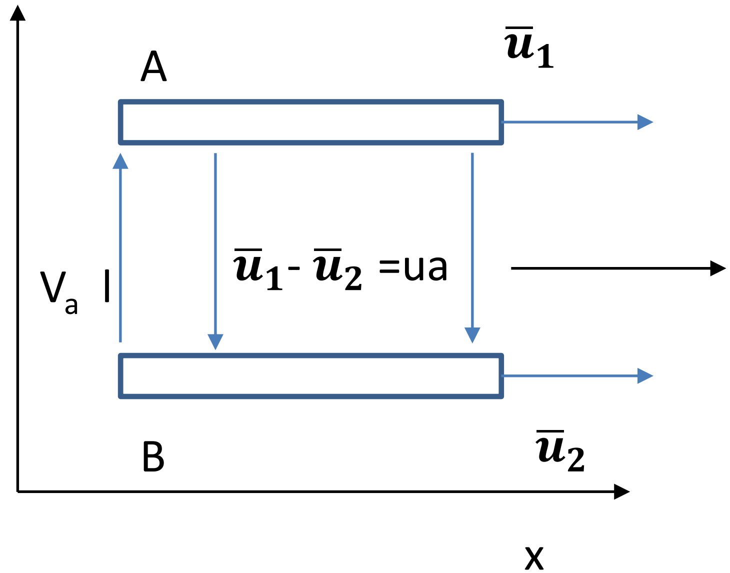

Fig.8:schematic representation of fluid layers separated by mixing length distance l

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Similarly in turbulent flow the fluctuationg velocity u’,v’ are responsible for turbulent shear and are the order of ua,va , thus Reynolds shear stress $\tau = - \rho \overline{v' u'}$ ……………………….(3)

Prandtl Mixing length theory

Mixing length l can be considered as transverse distance between two fluid layers such that the lumps of fluid mass from one layers could travel other layer and attached with them retaining its momentum along flow direction.

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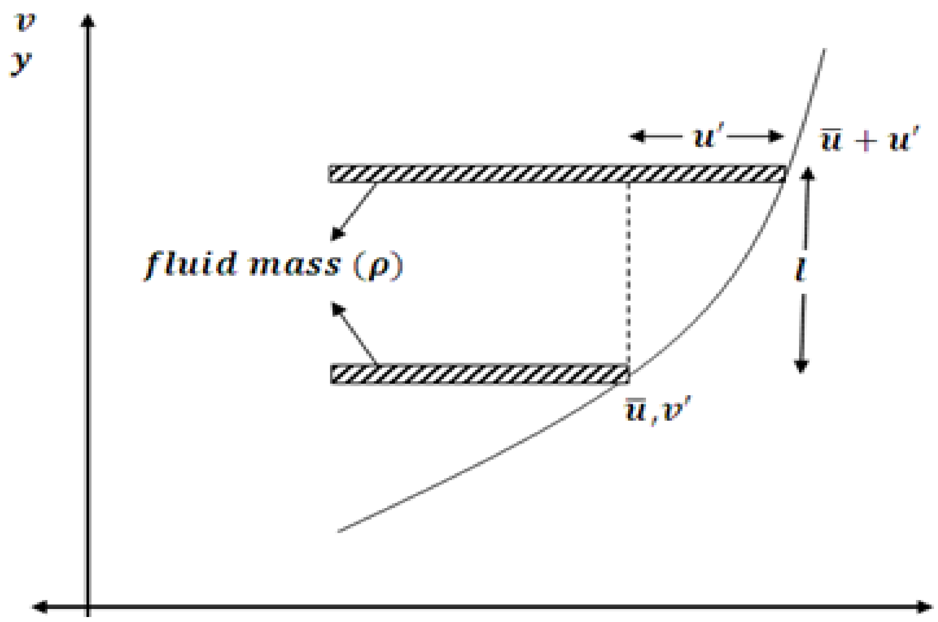

Fig.9: As can be seen in the figure,

$$u' = l\frac{du}{dy}$$

$$v' = l\frac{du}{dy}$$

$$\therefore u'v' = l^2\left(\frac{du}{dy}\right)^2$$

$\therefore$Shearstress$=\rho u'v' = \rho l^2\left(\frac{du}{dy}\right)^2$

Contd:

Total shear stress $\tau = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$

Velocity distribution in turbulent flow (Flow resistance equation)

Velocity distribution equation can be derived using relation of turbulent shear stress(Prandtl mixing length theory) and equation given by Nikuradse, i.e., l=ky where, k= von karman constant=0.4 (for details ,follow books)

$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

Or, du = $\frac{1}{k}\frac{dy}{y}\sqrt{\frac{\tau_o}{\rho}}$, where, l=ky and $\sqrt{\frac{\tau_o}{\rho}}$ = u* shear velocity

Contd:

On integration both side, we get

$u = \frac{u*}{K}$ loge (y)+C ……………………………………………………….(4)

Applying boundary condition , at y' ,u=0 ; y' is a small

distance from boundary up to which u=0

Finally we get equation (5)

$\frac{u}{u_*}$ = 5.75 log (y/y') ……………………………………………(5)

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Hydrodynamically smooth and rough boundary

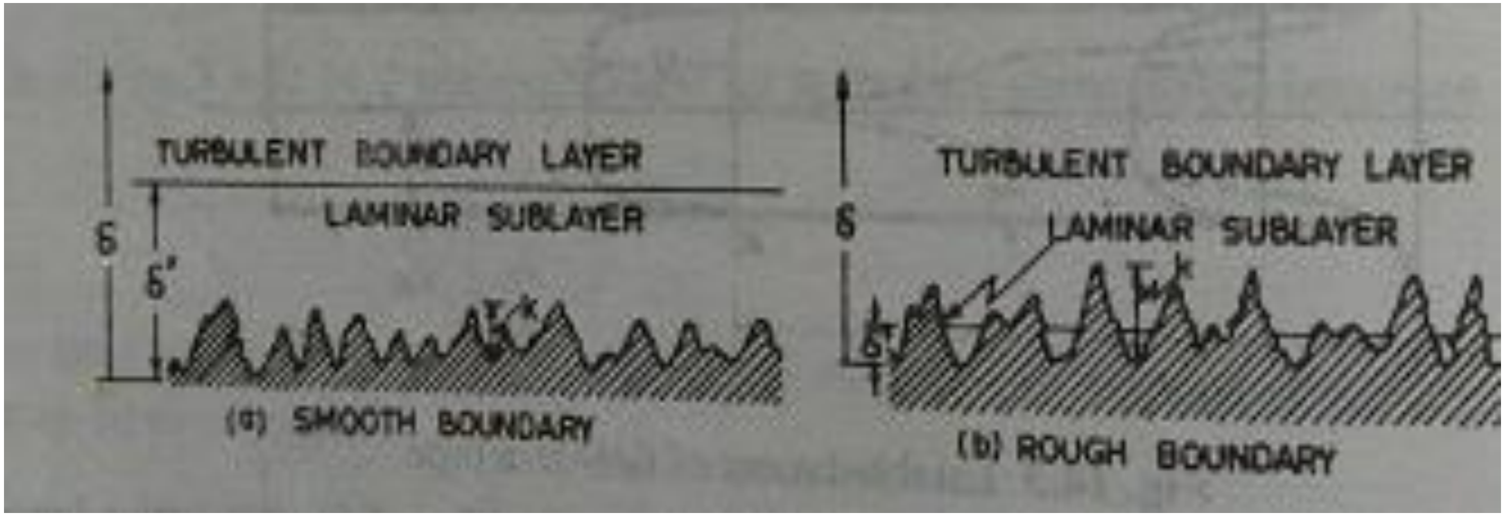


Fig.10:Schematic representation of smooth and rough boundary

For smooth surface ,$\frac{K_s}{\delta'} \leq 0.25$, and $\delta' = \frac{11.6\gamma}{u_*}$;

For rough surface $\frac{K_s}{\delta'} \geq 6.0$; for smooth boundary, y' = $\delta'/107$

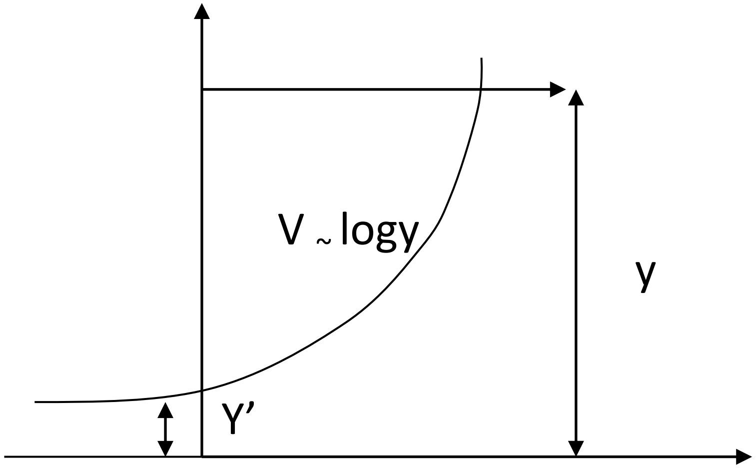
Where as for rough surface y'= Ks/30

Contd:

Now combining equation (5) with smooth and rough surface parameters, we get,

$\frac{u}{u_*} = 5.75 \log \left(\frac{y\, u_*}{\gamma}\right) + 5.50$ ....(6) (for smooth pipe)

$\frac{u}{u_*} = 5.75 \log \left(\frac{y}{Ks}\right) + 8.50$ ....(7)(for rough pipes)



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If we change the boundary condition to find the value of constant of integration, for example at center r=R, u= u_{max}, then e

u = $\frac{u*}{K}$ loge (y)+C ……………………………….(8)

C=u_{max} - $\frac{u*}{K}$ loge (R)

On substitution of this constant in above equation we get,

$$\frac{u-u_{max}}{u_*} = 2.5 \log_e \left(\frac{y}{R}\right)$$ ………………………….(9)

$$\frac{u-u_{max}}{u_*} = 5.75 \log_{10} \left(\frac{y}{R}\right)$$ …………………….(10)

Velocity distribution equation in terms of mean velocity(follow book for detail derivation)

For smooth boundary,

$$\frac{\bar{u}}{u_*} = 5.75 \log_{10} \left(\frac{u_* R}{v}\right) + 1.75$$ …………………………….(11)

Contd:

$\frac{\bar{u}}{u_*}$ = 5.75 $\log_{10}\left(\frac{R}{Ks}\right)$+ 4.75 ……………………(12) (for rough boundary)

Now on subtraction, eq(6)-(11) and eq.(7)-(12) , we get,

$\frac{u-\bar{u}}{u_*} = 5.75\ \log_{10}\left(\frac{y}{R}\right)$+ 3.75 ………………(13) ,which becomes independent of roughness parameter, can be applicable for rough and smooth surface both.

At center of pipe, y=R, thus eqn(13) becomes,

$\frac{u_{max}-\bar{u}}{u_*} = 3.75$ ……………………………………..(14)

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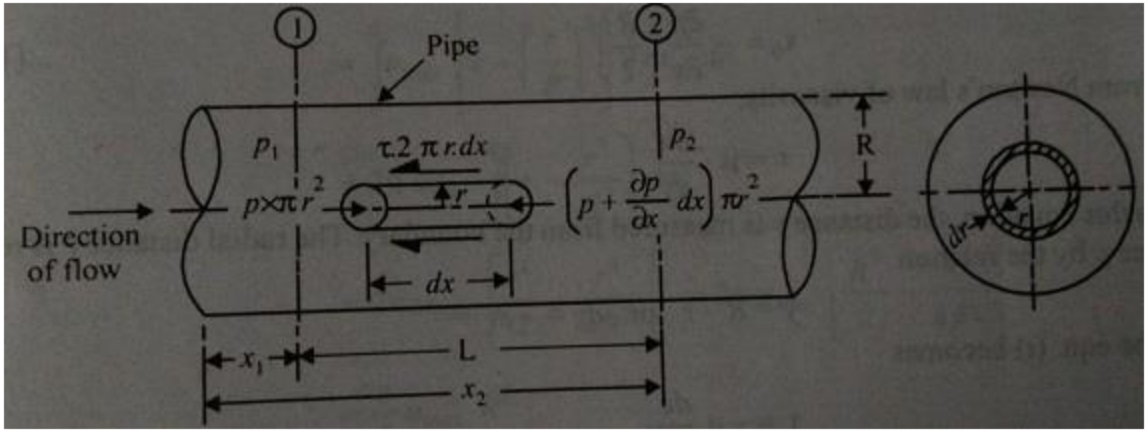
Loss of head due to friction in pipe, Darcy-Weisbach equation



From previous section of Hagen-Poiseuille equation derivation,

We have , $\tau_o = -\frac{dP}{dx} \times \frac{R}{2}$

or, $\tau_o = \frac{(P1-P2)}{l} \times \frac{D}{4}$ ....(15)

Dividing (15) both side by $\frac{\rho u^2}{2}$, we get

$\tau_o / \frac{\rho u^2}{2}$ = f= $(\frac{(P1-P2)}{l} \times \frac{D}{4}) / \frac{\rho u^2}{2}$, where f is dimensionless coefficient known as Darcy-Weisbach friction factor

Contd:

Finally we get,

$\frac{P_1 - P_2}{\gamma} = \frac{fl\rho u^2}{2gD\gamma}$, where $\gamma = \rho g$

$h_f = \frac{flu^2}{2gD}$ ....(16)

In case of coefficient of friction,use 4f in place of f

Also, $f = \frac{4\tau_o}{\frac{\rho u^2}{2}}$,or $\sqrt{\frac{8}{f}} = u\sqrt{\frac{\rho}{\tau_o}}$

But shear velocity $u_* = \sqrt{\frac{\tau_o}{\rho}}$, thus

$\sqrt{\frac{8}{f}} = \frac{u}{u_*}$ ....(17)

Contd:

Nikuradse Experiment

Nikuradse did dimensional analysis for pressure drop in uniform sand coated pipes in order to determine friction factor of pipe of different roughness in terms of uniform sand grain roughness height.

$\frac{\Delta P}{l} = \emptyset_1(u, D, k, \rho, \mu)$,here k= sand grain roughness height

Using Buckingham theorem and taking u,D, ρ as repeating variables, we get

$$\frac{\Delta P}{l} \times \frac{D}{\frac{\rho u^2}{2}} = f = \emptyset_2\left(\frac{\rho u D}{\mu}, \frac{K}{D}\right) \quad \text{.................(18)}$$

Thus,

$f = \emptyset_2\left(\frac{\rho u D}{\mu}, \frac{K}{D}\right)$, some times k/D is written in terms of relative smoothness as R/k

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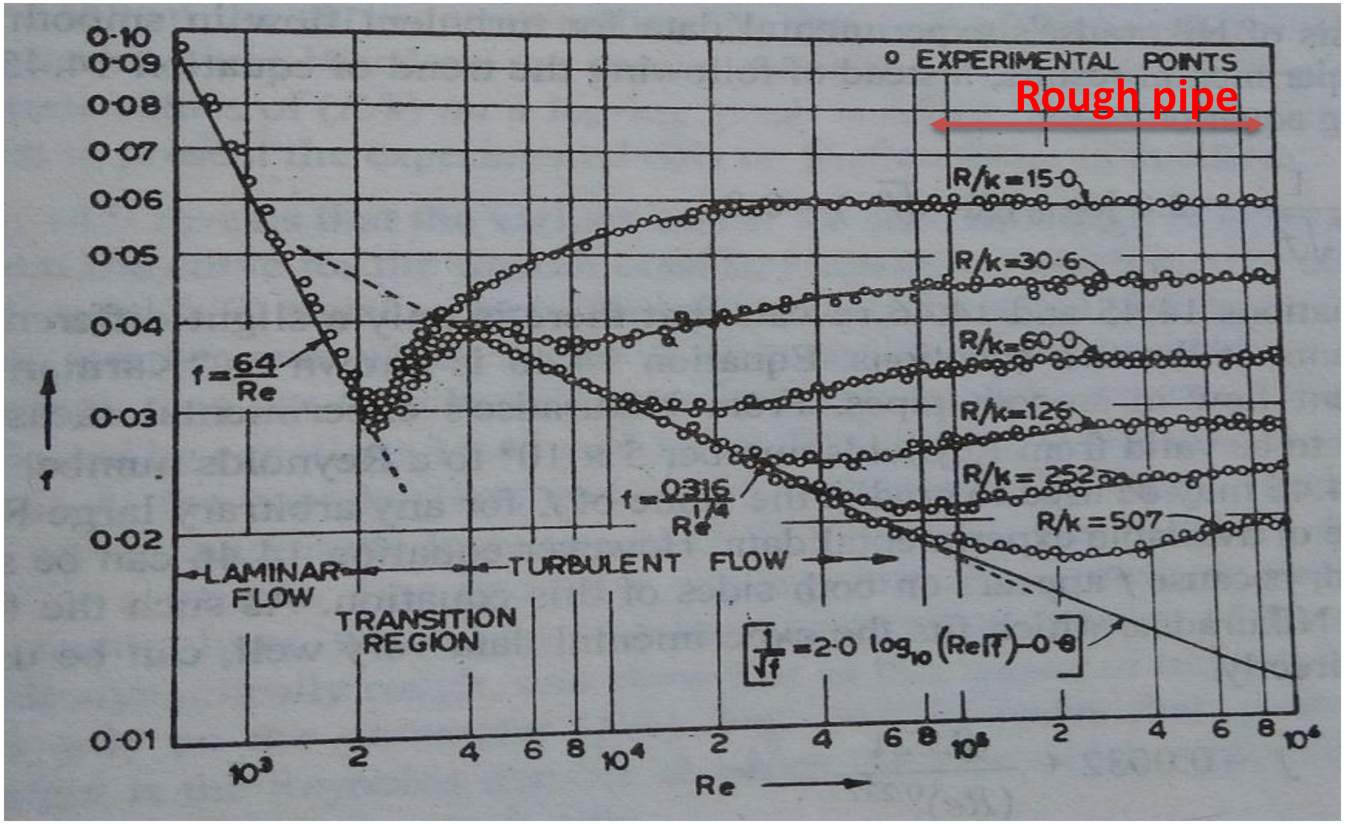
Nikuradse varies pipe D, u,k and measured head loss in sand coated pipe of constant length and then computed friction factor f. The entire available data were plotted Stanton in log scale .

Fig.11



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Following findings were noted from his graph,

- For smooth surface friction factor becomes independent of K/D or R/K and depends on Reynolds number Re.
- On the other hand for rough boundary f becomes independent of Re and depends on K/D or R/k
- For laminar flow region f= 64/Re , up to Re=2000
- There exist no specific relationship for 2000 <Re <4000 (Transition region)
- For fully developed turbulent flow f friction factor depends on both Re and K/D or R/K
- Blasius has given equation for smooth pipes in turbulent flow as $f = \frac{0.316}{Re^{1/6}}$ ....(19) it is valid up to range $4x10^3$ to 10^5 , after this there is apparent deviation of points from straight line.

Contd:

- For Reynolds number exceeding 10^5 can be obtained from logarithmic law of velocity distribution for smooth and rough pipes as,

For smooth pipe, from equation (11)

$$\frac{\bar{u}}{u_*} = 5.75 \log_{10}\left(\frac{u_* R}{v}\right) + 1.75$$

Put $u_* = u\sqrt{\frac{f}{8}}$ and R= D/2 in above equation , it becomes

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.88 \quad \text{....................(20)}$$

Here the constant 0.88 slightly vary from lines obtained from graph So, in order to fit exact with graph lines equation (20) is written as

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8 \quad \text{........................(21)} \quad \text{(for smooth pipes)}$$

Contd:

Equation(21) is valid for Re = $5x10^4$ to $4x\ 10^7$

Similarly for rough pipe, from equation(12)

$$\frac{\bar{u}}{u_*} = 5.75 \log_{10}\left(\frac{R}{Ks}\right) + 4.75$$

Put $u_* = u\sqrt{\frac{f}{8}}$ in above equation

$$\frac{1}{\sqrt{f}} = 2\log_{10}(R/\mathrm{Ks}) + 1.68$$

In order to exact fit in to data ,the constant 1.68 was replaced with 1.75

$$\frac{1}{\sqrt{f}} = 2\log_{10}(R/\mathrm{Ks}) + 1.75 \text{ ....(22) (for rough pipe)}$$

Which is independent of Reynolds number depicted by several horizontal lines for different values of R/Ks

Contd:

Equation (21) and (22) may be rearranged by subtracting $2\log_{10}$ (R/Ks) from(21) and (22)

We get,

$\frac{1}{\sqrt{f}}$ - $2\log_{10}\frac{R}{K}$ = $2\log_{10}\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ - 0.8 ....(23) for smooth surface

$\frac{1}{\sqrt{f}}$ - $2\log_{10}\frac{R}{K}$ = 1.75 ....(24) for rough surface

Equation (23) shows that the left hand side term $\frac{1}{\sqrt{f}}$ - $2\log_{10}\frac{R}{K}$ is function of $\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ in smooth case, for rough it becomes constant,1.75.

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A plot of graph using Nikuradse data was used in above equation , it is shown below in graph

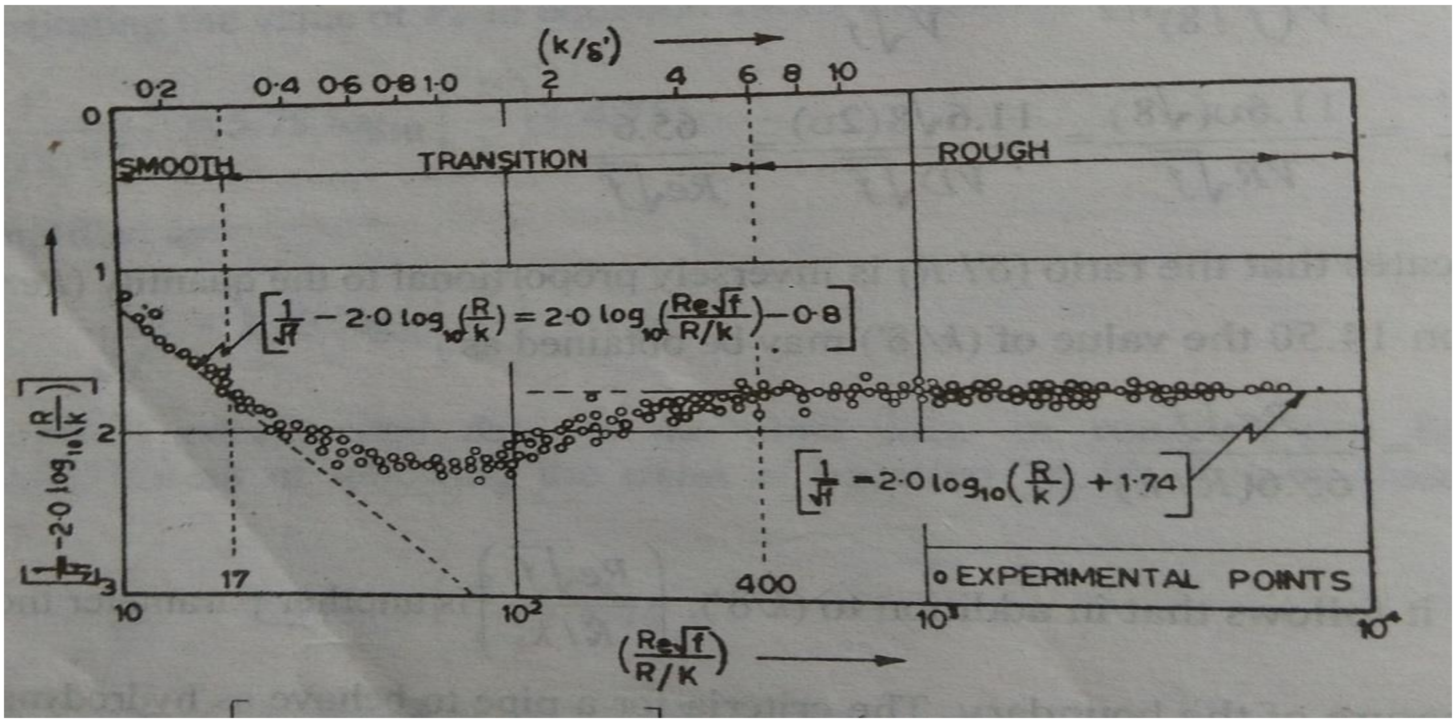

Fig.12: Plot of $\frac{1}{\sqrt{f}}$ - 2$\log_{10}\frac{R}{K}$ vs $\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ *for artificially rough pipe*

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From above chart , it is observed that,

$\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ < 17 implies hydrodynamically smooth pipe

$\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ >400 , implies , hydrodynamically rough pipe

17< $\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ < 400 , implies , behave as transition

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Variation of friction factor for commercial pipes

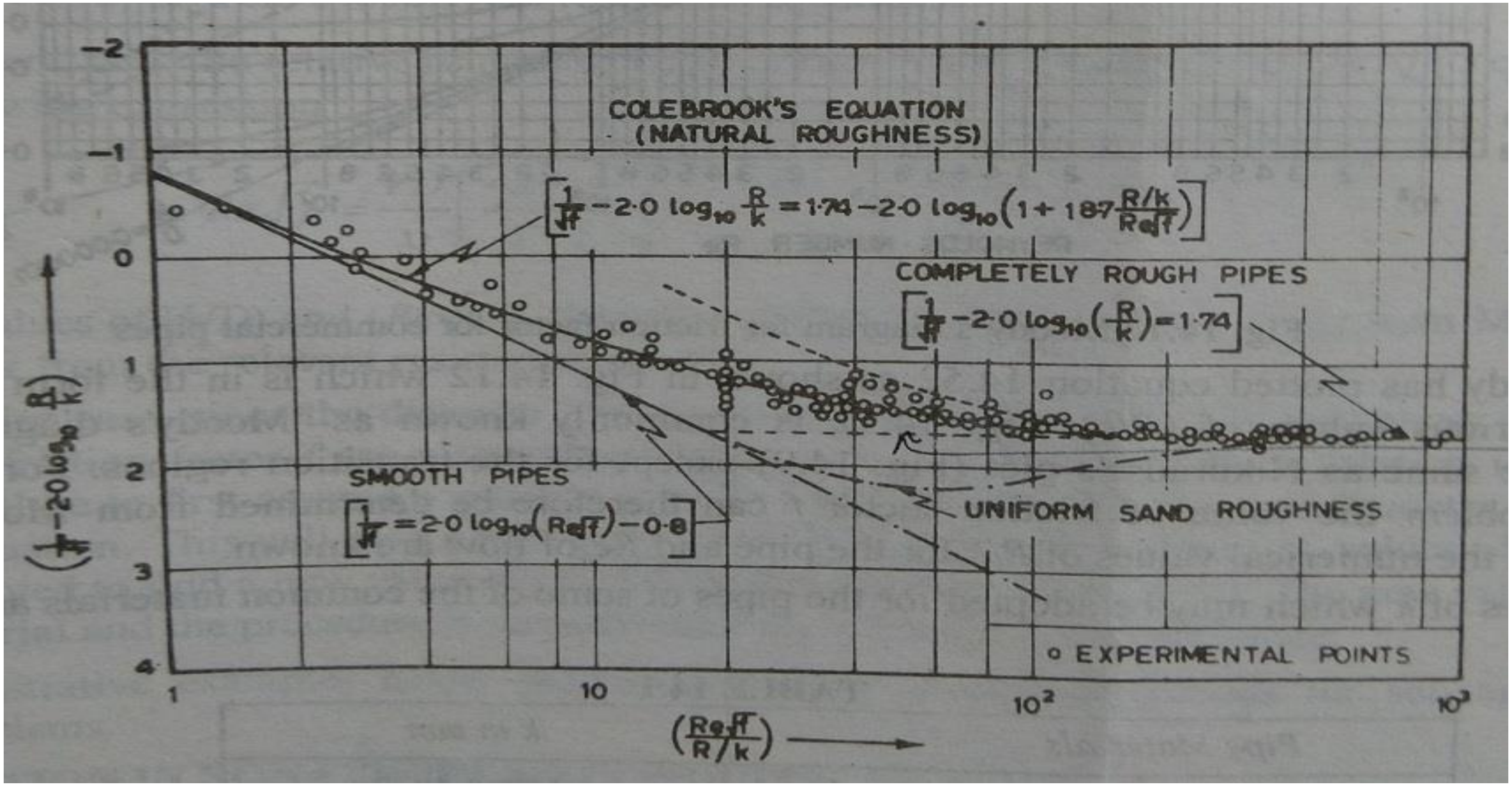

Fig.13: Plot of{ $\frac{1}{\sqrt{f}}$ - $2\log_{10}\frac{R}{K}$ } vs $\left(\frac{Re\sqrt{f}}{\frac{R}{K}}\right)$ *for commercial pipe*

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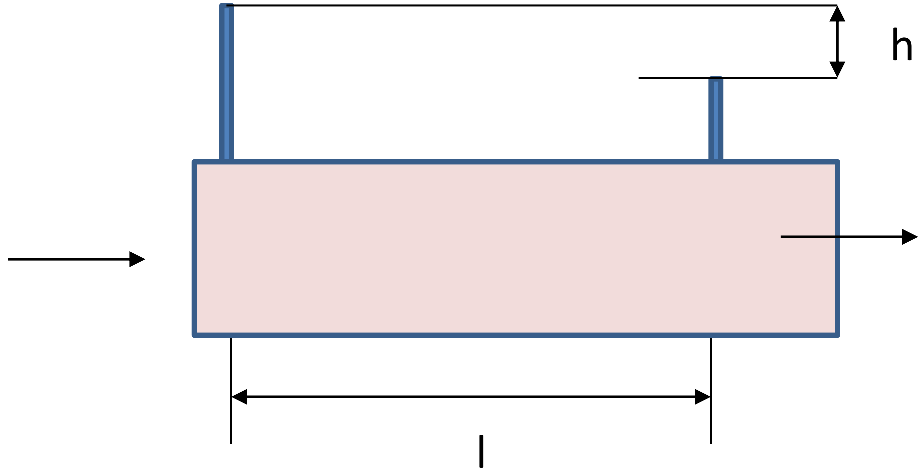
Nikuradse experimental graphical results is for uniform sand grain coated pipes, it can not be directly applied for commercial pipes to evaluate friction factor .

So, in order to use Nikuradse result, friction factor f for commercial pipe $f = 2gdh/lu^2$
Here h= can be measured from piezometer



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The obtained friction factor for commercial pipes fc must be equal to sand grain coated pipes for particular roughness k

fc = fs….

the value of ks obtained from solving equation

$\frac{1}{\sqrt{f}} = 2\log_{10}(R/\mathrm{Ks}) + 1.75$, this ks will be the final equivalent roughness of commercial pipes at high Reynolds number

However, in transition region of Reynolds number, the two values obtained for sand grain coated pipes and commercial pipes slightly deviates.

Finally Colebrook-White developed a common equation for best fit line after plotting all the data from experiment as shown in figure 13.

Contd:

Colebrook-White equation,

$$\frac{1}{\sqrt{f}} - 2\log_{10}\frac{R}{K} = 1.74 - 2.0\log_{10}\left(1 + 18.7\frac{\frac{R}{K}}{Re\sqrt{f}}\right) \quad \ldots\ldots (25)$$

Simplified form of Colebrook-White equation

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Swami-jain formula

$$f = \frac{0.25}{\left(\log_{10}\left(\frac{k/D}{3.7} + \frac{5.74}{Re\hat{}0.9}\right)\right)\hat{}2}$$

Salient features of Moody's chart

L.F. Moody plotted equation (25) which of the form f versus Re for various values of (R/k) as shown in fig. 14

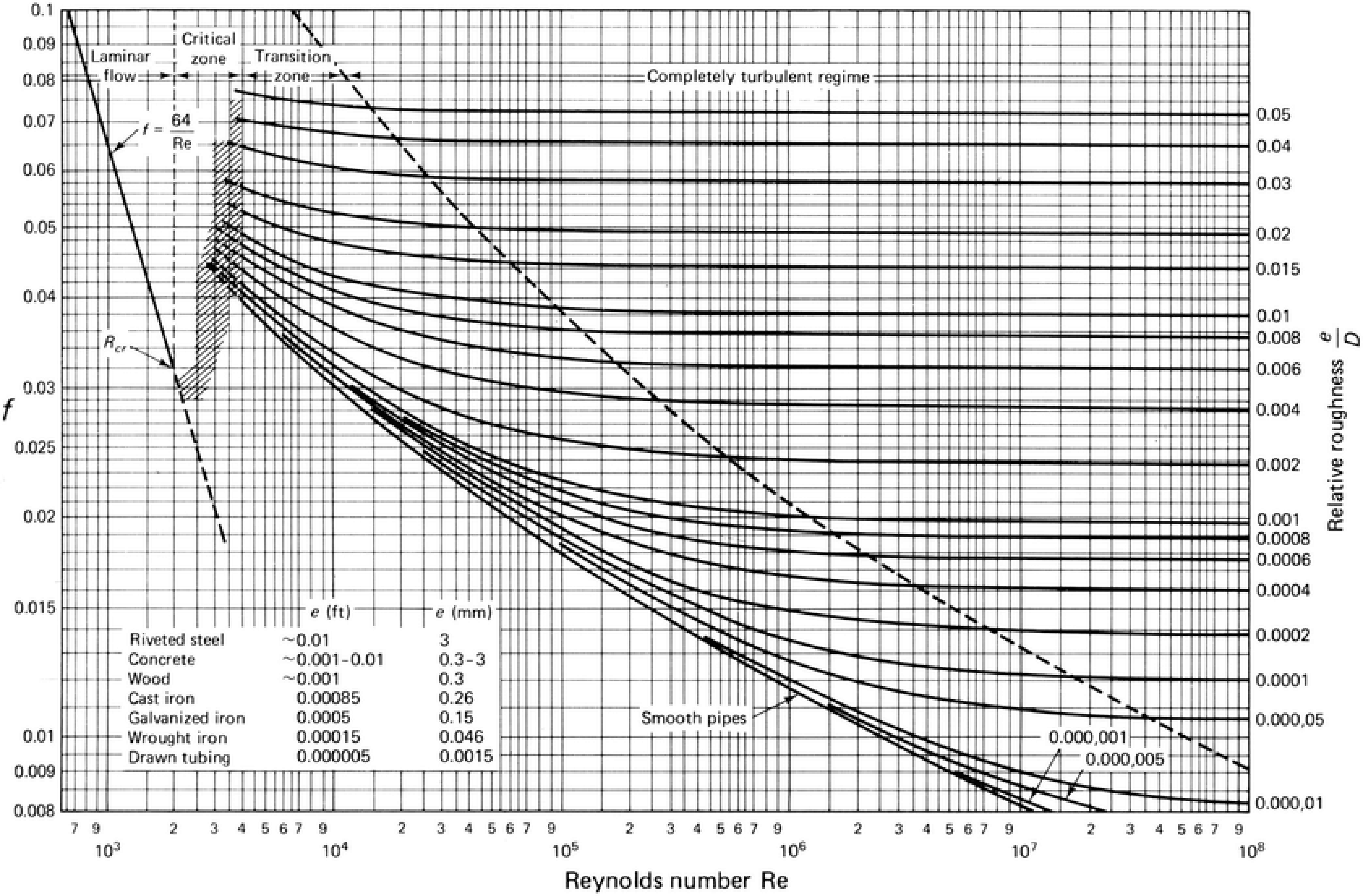


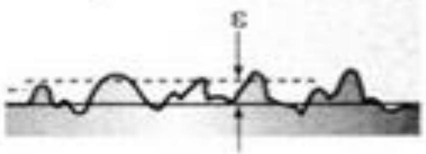
Fig.14: Moody’s chart

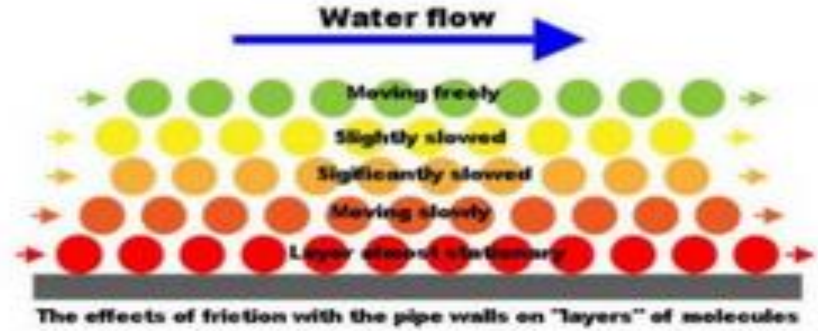
Contd:

Head Loss in Pipes

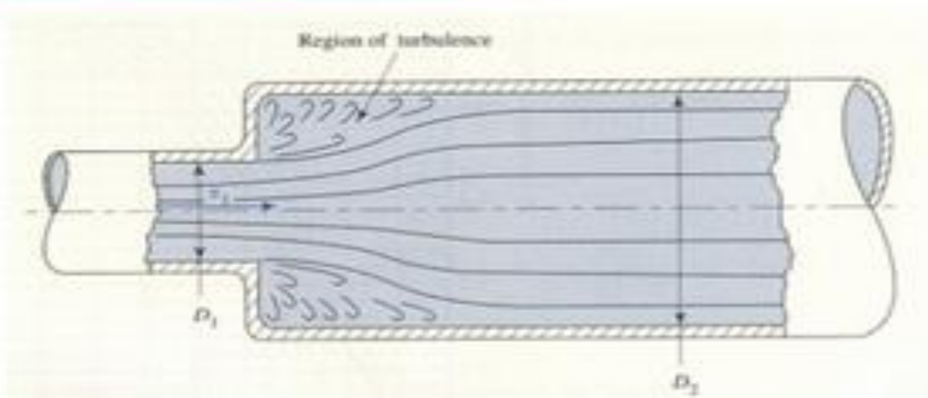
- Total Head Loss=Major Losses+ Minor Losses

- **Major Loss:** Due to pipe friction



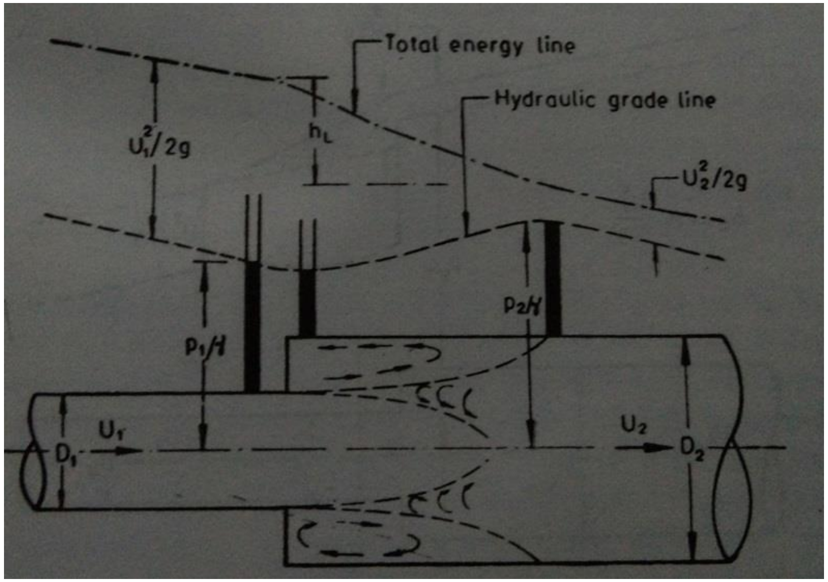


- **Minor Loss:** Due to pipe fittings, bents and valves etc

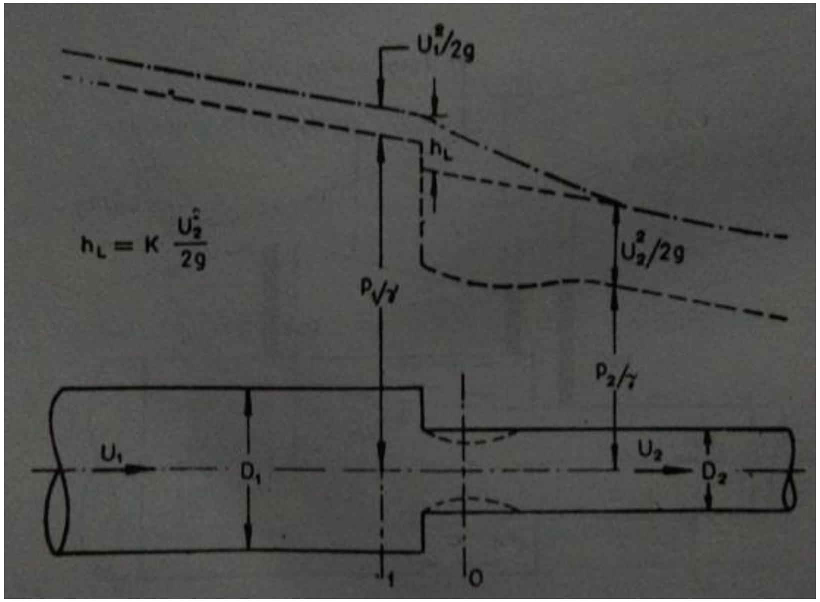


Contd:

Sudden expansion



Sudden contraction



a) Sudden expansion loss hl = $\frac{(v1-v2)^2}{2g}$

b) Sudden contraction hl = $0.5\frac{v^2}{2g}$

c) Entrance loss = $0.5\frac{v^2}{2g}$

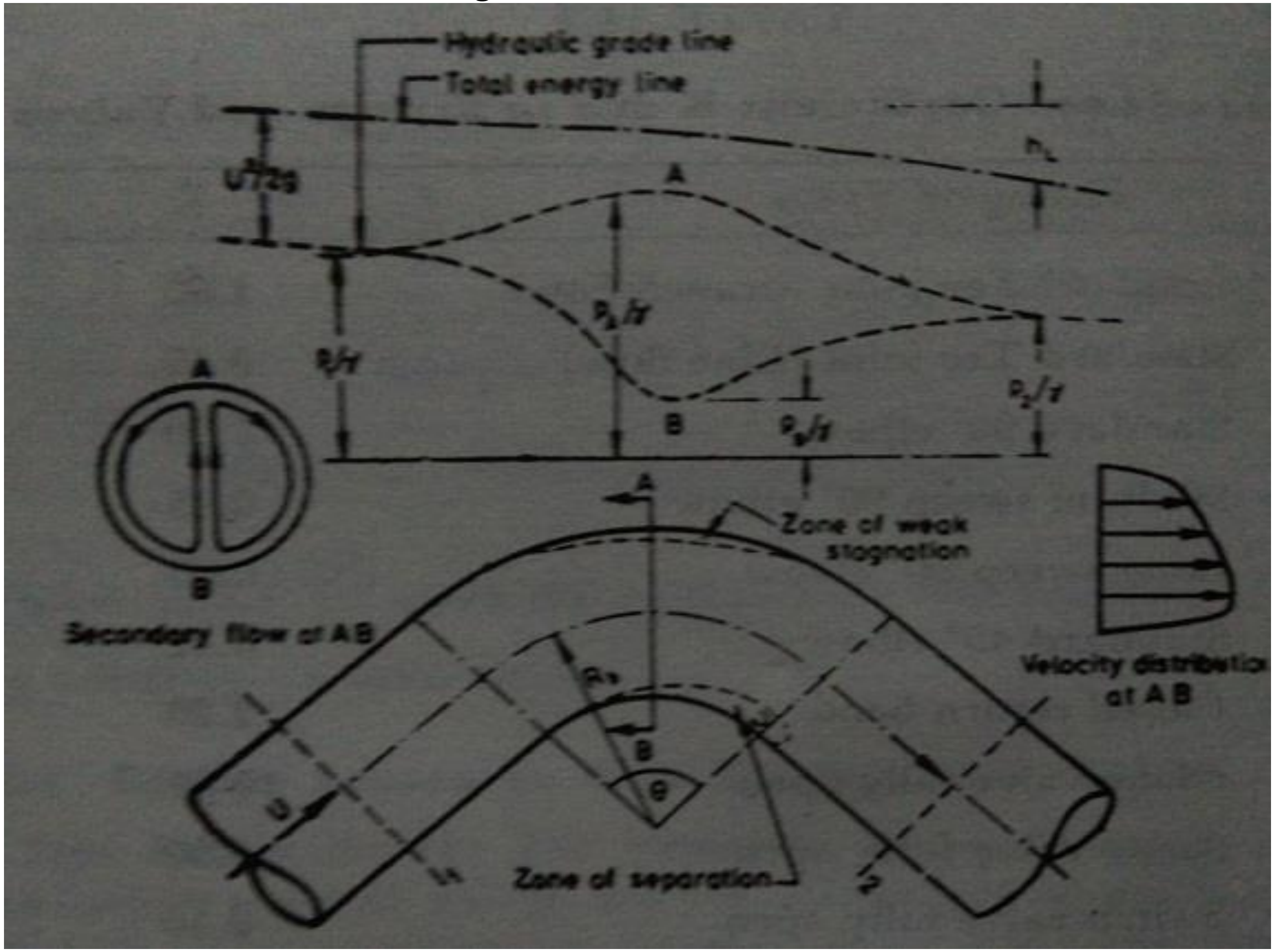
Contd:

d) Exit loss = $\frac{v^2}{2g}$

e) Gradual contraction or expansion loss = k $\frac{(v1-v2)^2}{2g}$

f) Bend loss = k $\frac{v^2}{2g}$

g) Fitting loss = k $\frac{v^2}{2g}$



Contd:

Pipe flow problems

Deal with six variables in pipe flow problem

i.e., μ,l,D,k,Q,h_f

μ,k and l are always known so 3 remaining variables to be found out

So, three types of problem

Known equation is, $h_f = \frac{flu^2}{2gD} = \frac{8flQ^2}{\pi^2 gD^5}$, f from Moody's chart for Re and K/D

a) Compute h_f for known μ,l,D,k,Q

determine u, Re, and k/D ; use Moody's chart for friction factor, for this compute h_f

b) Compute D for known μ,l,k,Q,h_f

assume friction factor f, determine D using D-Weisbach formula, find Re, k/D , again find f from chart and tally with assumed f. if its same then stop otherwise go for next trial

Contd:

C)Determine Q for known μ,l,D,k,h_f

Other problems

- Pipes in series and parallel
- Equivalent length of pipe
- Head loss in parallel and in series
- Three reservoir problem
- Syphon problem