Open Channel Hydraulics

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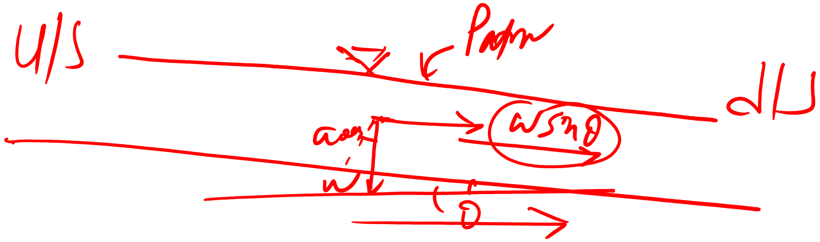
CONTENTS

❑ Basics of Open channel/Free surface flow

- Introduction, practical application and difference between pipe and open channel flow
- Classification(boundary, geometry, shape)
- Geometrics properties
- Classification of flow in open channel

❑ Open channel is a conduit in which liquid flows with a free a free surface under gravity.



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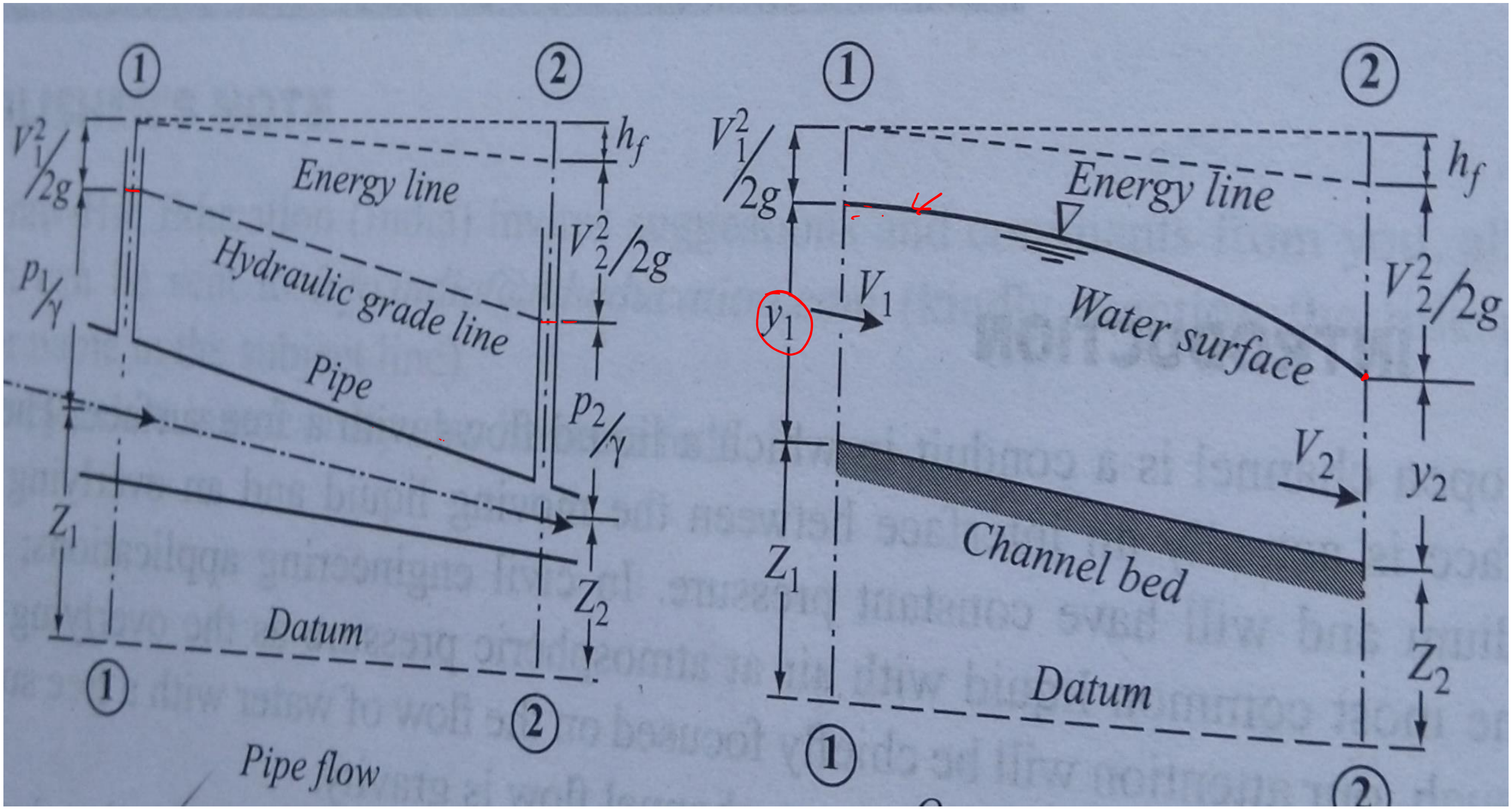

Fig. 2: Schematic representation of Pipe flow and Free surface flow(open channel)

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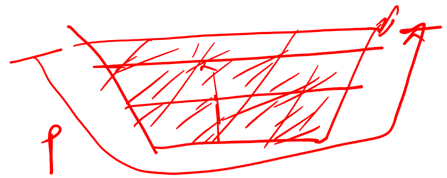

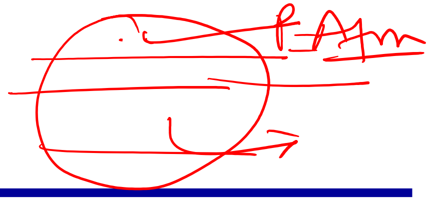

Pipe flow	Open channel flow
No free surface, Pipe always runs full, presence of gauge pressure	Presence of free surface , exposed to atmospheric pressure
Flow due to Energy gradient,	Flows due to potential energy gradient. i.e., under gravity
HGL = Piezometric line	HGL coincides with water surface
The flow takes place in prefixed section	As Flow parameter changes, wetted x section area also changes
Fixed Geometry of pipe	Geometry may vary, mobile boundary channel
Boundary roughness is in reasonable order, low mostly of the order of $k_s/\delta' \leq 0.25$	Boundary roughness vary in wide range , mostly of the order of $k_s/\delta' \geq 6$
Flow is governed by Reynolds number as the prime force is viscous force	Flow is governed by Froude number as the prime force is Gravity force

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Practical application

- It is a means of water transport
- For example Canal is used for Irrigation, drinking water supply
- Power canal, Drains, Sewer canal
- Navigation channel, Migration and survival of Aquatic lives, Channel is used for conveyance of sand , boulder etc.

Channel: Channel is a conduit made by nature it self to transport water. i.e., discharge has power to make channel shape and slope in order to flow downstream under the influence of gravity.

Canal: All manmade conduits for water transport , Surface exposed to atmosphere, free surface flow

Here we include canal into Channel for their hydraulic studies purpose.

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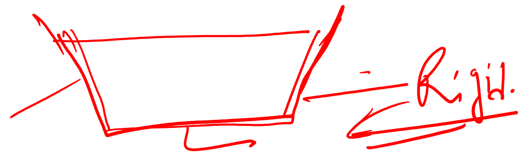

Classification :

<u>Rigid Boundary channel</u>: whose beds and banks are fixed.
For example : Power canal, Water supply canal etc.

<u>Mobile Boundary Channel :</u> The channel whose boundary changes.
For example : all natural channels i.e., rivers, streams etc.

<u>Prismatic channel</u>: Whose x-section and slope doesn't vary

<u>Non-prismatic channel:</u> Whose x-section and slope varies

<u>Natural :</u> Channel
<u>Artificial :</u> Canal

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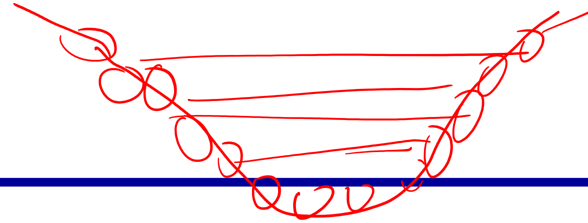

Some Photographs:

Fig.2.: (1) Aerial view of Bheri river (Jajarkot) as Mobile boundary channels in gravel bed streams. Photographs (2) depicts same river as gravel bed channels whose banks are rocky .

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Fig.3 & 4: Photographs 3 and 4 (Kamla river ,Nepal) shows alluvial channel as mobile boundary channel

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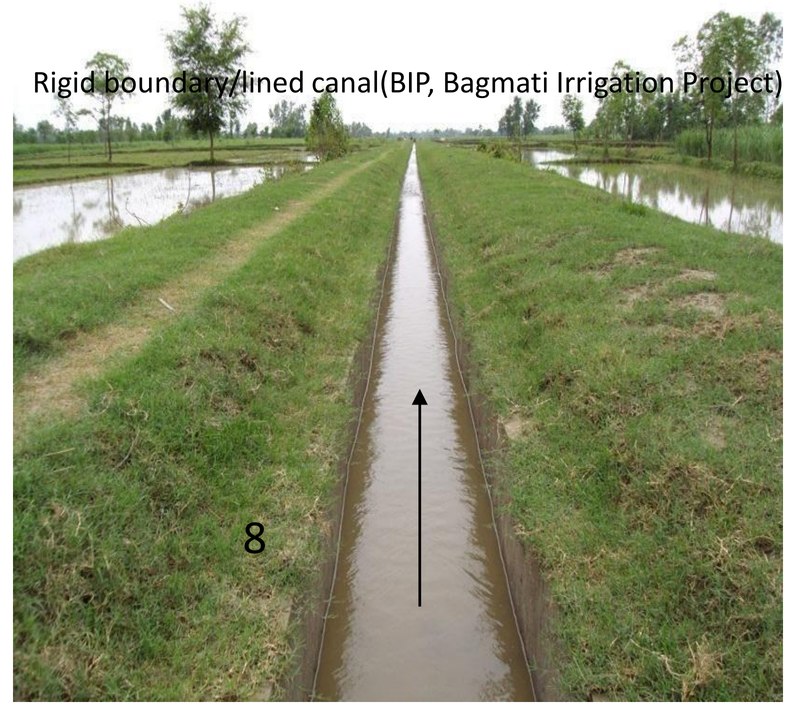
Fig.5 & 6: Photographs 5 is of Pakhar khola (Sindhupalchowk district) , boulder stream and figure 6 shows power canal of Sunkoshi HPP as Prismatic channel


Earthen canal(BIP, Bagmati Irrigation Project)
7


Rigid boundary/lined canal(BIP, Bagmati Irrigation Project)
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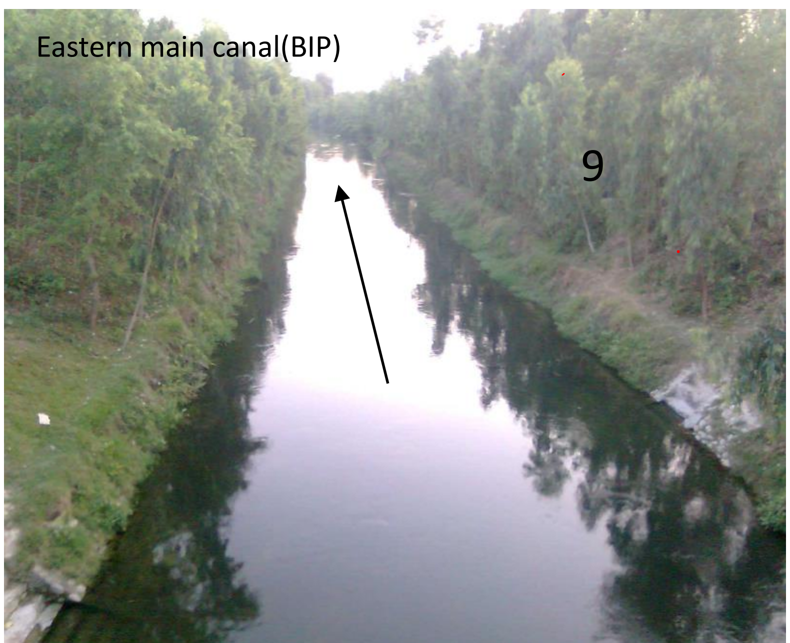
Eastern main canal(BIP)
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Eastern main canal(BIP)

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Fig. 11: Upper Ganga canal passes through Roorkee, India for Irrigation and drinking water supply, Irrigates agricultural land of UP and supplies water to Delhi for drinking

Qd = 250 (irrigation for 20 lakh hectares) + 50 (drinking water) = 300 m3/s

Total Length= 292 Km

Fig 12: Navigation Channel

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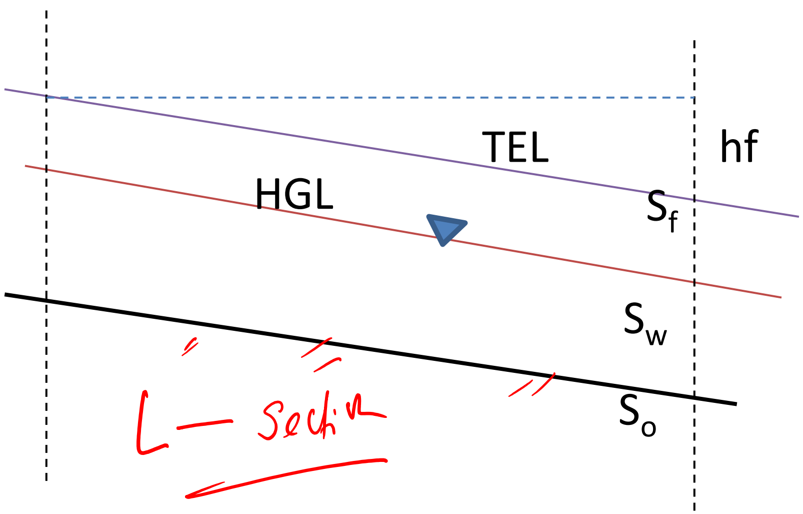
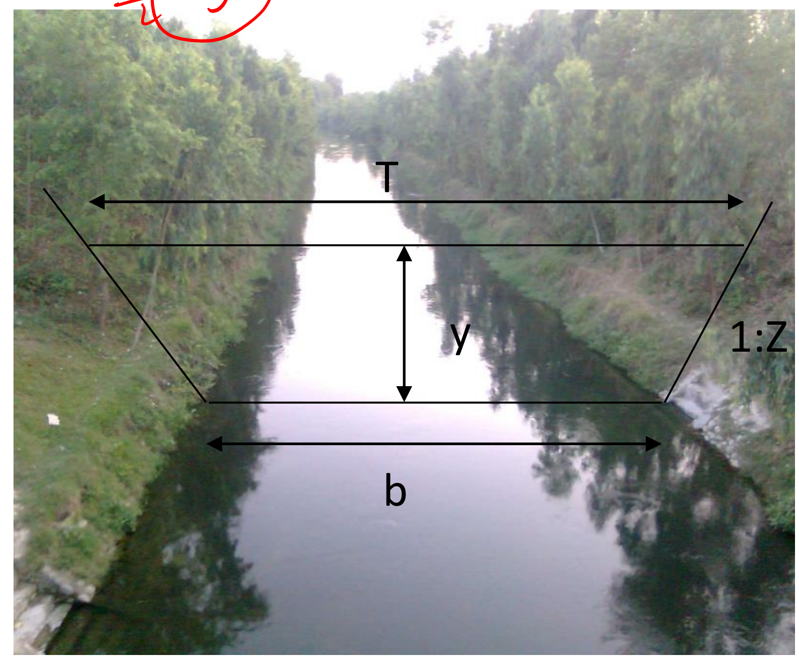
<u>Geometric Properties</u>

Let us consider a Simple Section such as trapezoidal section of a canal



L – Section



X – Section

Flowing area A= by+zy^2

Top width = b+zy+zy = b+2zy

Wetted perimeter P = b+ 2y $\sqrt{1+z^2}$

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Hydraulic radius = A/P = $(by+zy^2)/(b+2y\sqrt{1+z^2})$

Hydraulic mean depth = A/T = $(by+zy^2)/(b+2zy)$

- **Hydraulic Radius (R_h) or Hydraulic Diameter:** It is the ratio of area of flow to wetted perimeter of a channel or pipe

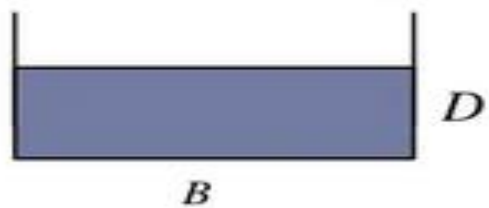
$$R_h = \frac{Area}{wetted\ perimeter} = \frac{A}{P}$$

For Circular Pipe

$$R_h = \frac{A}{P} = \frac{((\pi/4)D^2)}{\pi D} = \frac{D}{4}$$

$$D = 4R_h$$

For Rectangular pipe



$$R_h = \frac{A}{P} = \frac{BD}{B+2D}$$

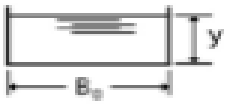
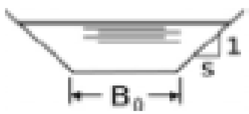
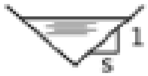
$$R_h = \frac{VD}{\nu} = \frac{4VR_h}{\nu}$$

By replacing D with R_h, Reynolds' number formulae can be used for non-circular sections as well.

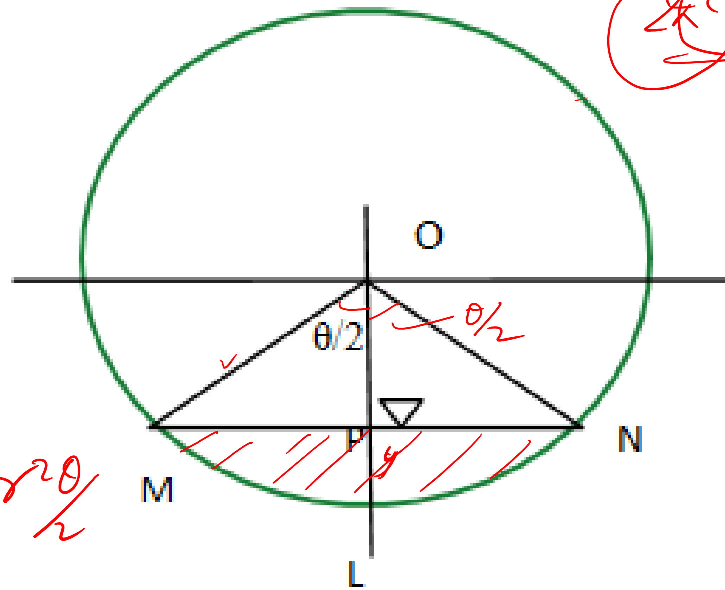
4

Note: hydraulic Radius gives us indication for most economical section. More the Rh more economical will be the section.

Table 1-1. Properties of typical channel cross sections

Area, A	Wetted Perimeter, P	Hydraulic radius, R	Top width, B	Hydraulic depth, D	
$B_o y$	$B_o + 2y$	$\frac{B_o y}{B_o + 2y}$	B_o	y	
$(B_o + sy)y$	$B_o + 2y\sqrt{1+s^2}$	$\frac{(B_o + sy)y}{B_o + 2y\sqrt{1+s^2}}$	$B_o + 2sy$	$\frac{(B_o + sy)y}{B_o + 2sy}$	
sy^2	$2y\sqrt{1+s^2}$	$\frac{sy}{2\sqrt{1+s^2}}$	$2sy$	$0.5y$	
$\frac{1}{8}(\theta - \sin\theta)D_o^2$	$\frac{1}{2}\theta D_o$	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right) D_o$	$D_o \sin\frac{1}{2}\theta$	$\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right)\frac{D_o}{8}$	

Circular channel



Rough , for 2pi radian = pi* r^2

Thus for $\theta = r^2\theta/2$

2pi= 2pi r

$\theta = r\theta$

Let θ be angle subtended by water level MN at center of circle. And PL= y be water depth in channel

Area of flowing section , MLN= OMLN-OMN (triangle) = $r^2\theta/2$ - 1/2 (2MP * OP)

$= r^2\theta/2 - 1/2\,(2\ \ OM\ Sin\theta/2\ OM\ cos\theta/2\ )$

$= r^2\theta/2 - 1/2\,(2\,r\,Sin\theta/2\ \ cos\theta/2)$

$= r^2\,(\theta - sin\theta)/2$

$$A = \frac{D^2}{8}(\theta - sin\theta)$$

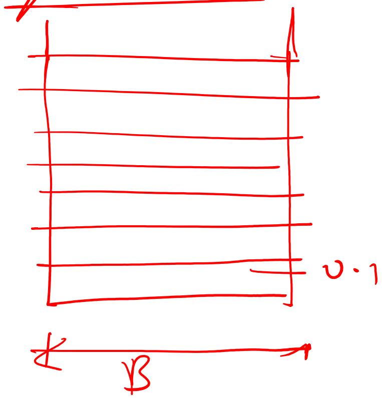
Similarly wetted perimeter, MLN = $r\theta = D\theta/2$

and also $cos\theta/2 = OP/r = (OL-PL)/r = (r-y)/r = ((D/2)-y)/(D/2) = (1-2y/D)$

Rectangular deep channel

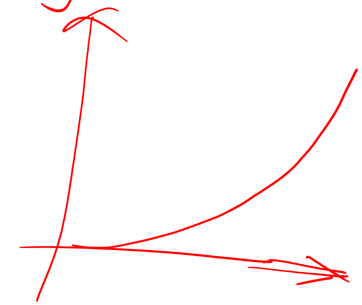
$y >> B$

$B << y$



$R = A/P = \frac{by}{b+2y}$

$\frac{y}{2y_0}$ | $\frac{R}{R_0}$



Triangular Channel ✓

Trapezoidal ✓

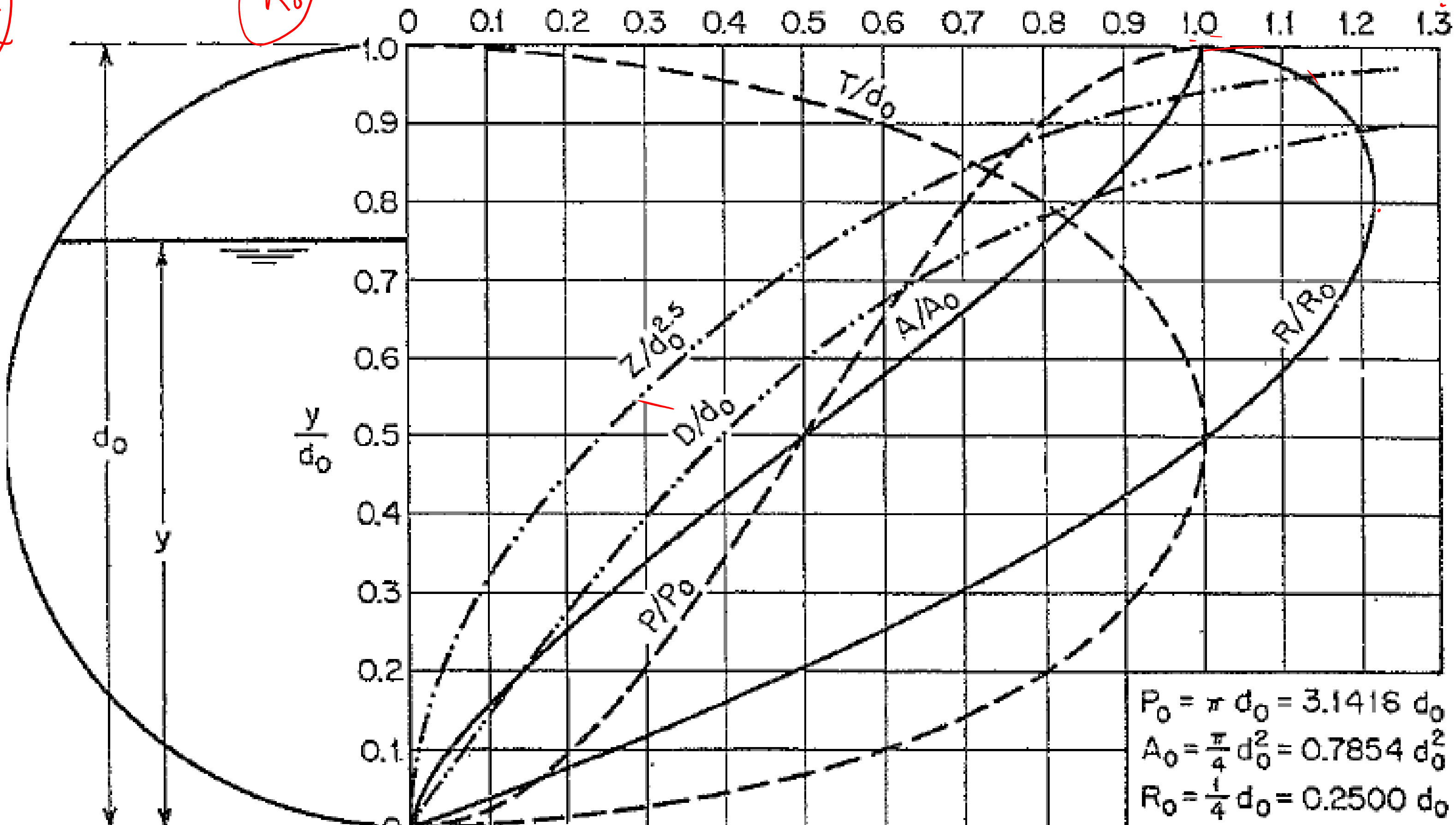


FIG. 2-1. Geometric elements of a circular section.

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S_o = Bed slope of channel
S_w = water surface slope
S_f = slope of TEL
At uniform flow, $S_o = S_w = S_f$

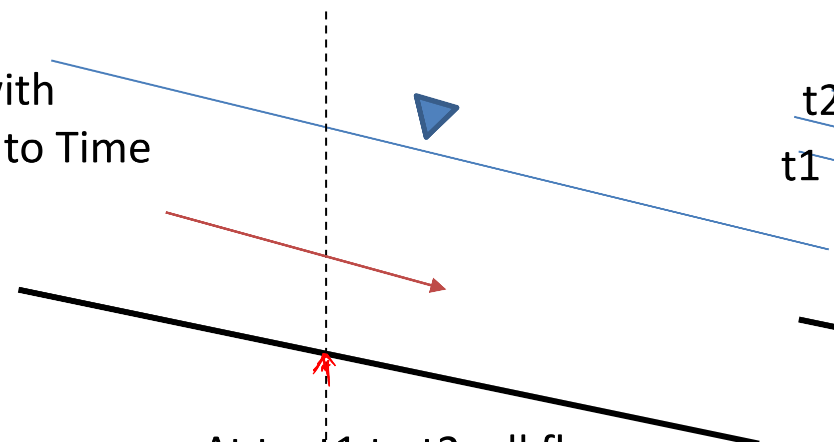
<u>Flow Classification</u>

- Steady and Unsteady flow
- Uniform and Nonuniform flow
- Gradually and Rapidly varied flow
- Super critical , critical and sub critical flow

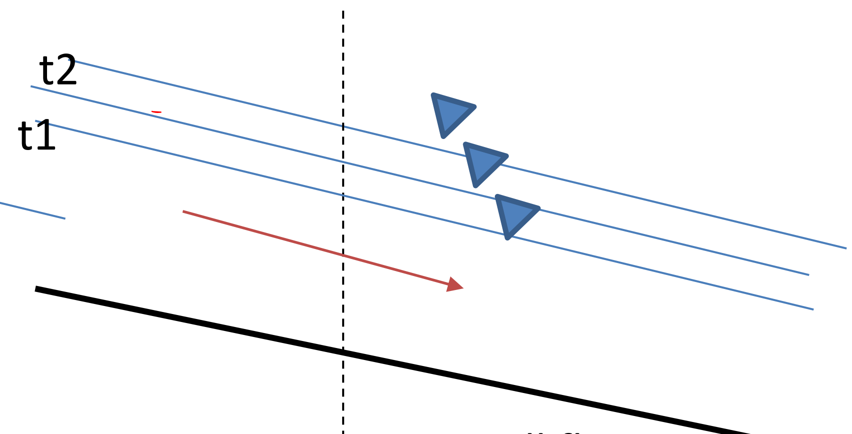
Contd:

Steady	and Unsteady flow
$\left(\frac{\partial u}{\partial t}\right)_{t=t1\ to\ t2} = 0$	$\left(\frac{\partial u}{\partial t}\right)_{t=t1\ to\ t2} \neq 0$
$\left(\frac{\partial Q}{\partial t}\right)_{t=t1\ to\ t2} = 0$	$\left(\frac{\partial Q}{\partial t}\right)_{t=t1\ to\ t2} \neq 0$
$\left(\frac{\partial y}{\partial t}\right)_{t=t1\ to\ t2} = 0$	$\left(\frac{\partial y}{\partial t}\right)_{t=t1\ to\ t2} \neq 0$

Study with respect to Time



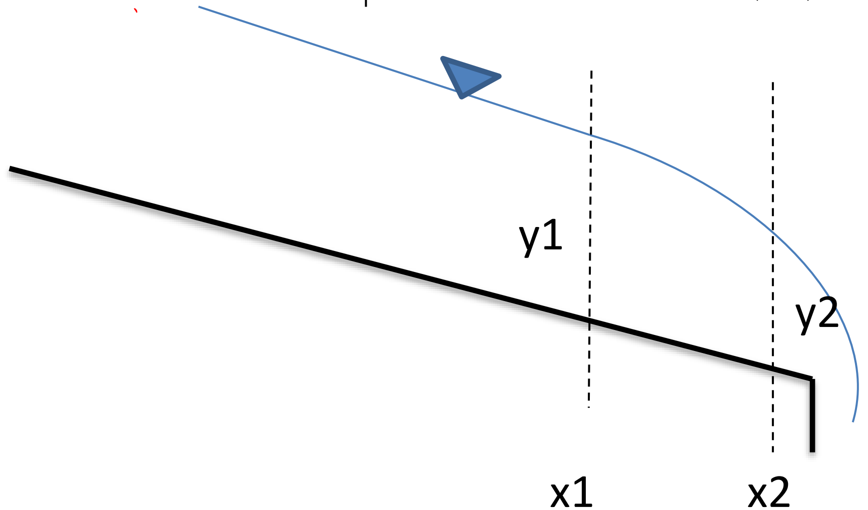
At t = t1 to t2, all flow parameter remains constant w.r.t time



At t = t1 to t2, all flow parameter varies w.r.t time

Contd:

Uniform	and	Non-uniform flow
$\left(\frac{\partial u}{\partial x}\right)_{x=x1 \text{ to } x=x2} = 0$		$\left(\frac{\partial u}{\partial x}\right)_{x=x1 \text{ to } x=x2} \neq 0$
$\left(\frac{\partial Q}{\partial x}\right)_{x=x1 \text{ to } x=x2} = 0$		$\left(\frac{\partial Q}{\partial x}\right)_{x=x1 \text{ to } x=x2} \neq 0$
$\left(\frac{\partial y}{\partial x}\right)_{x=x1 \text{ to } x=x2} = 0$		$\left(\frac{\partial y}{\partial x}\right)_{x=x1 \text{ to } x=x2} \neq 0$



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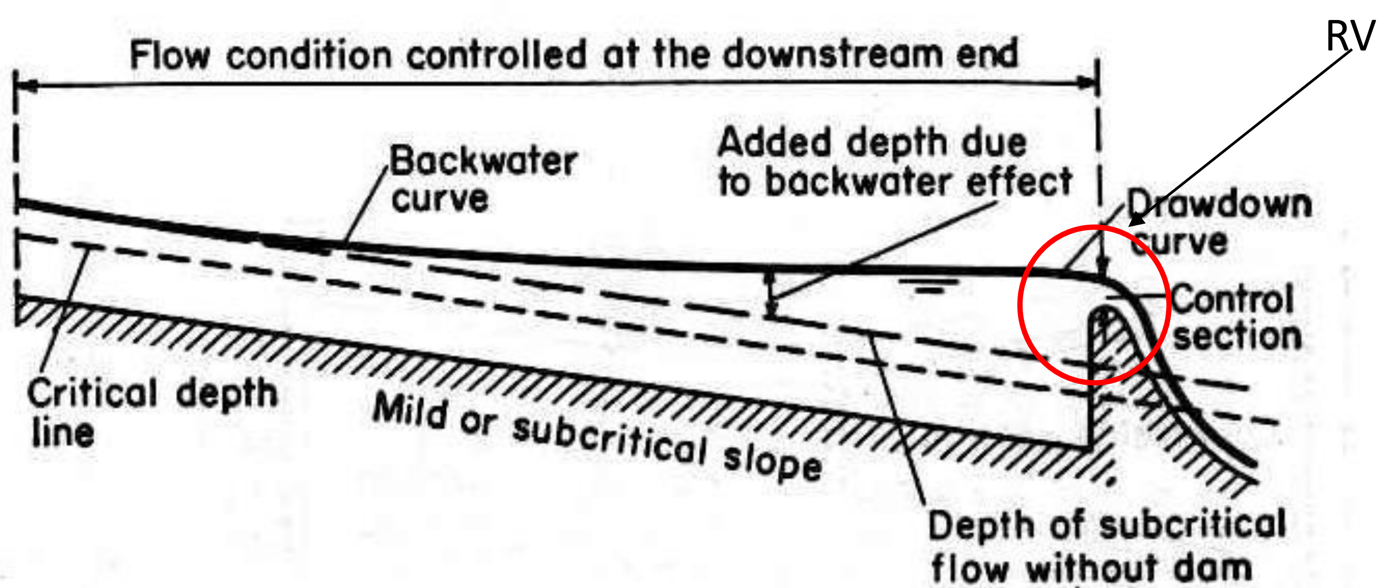
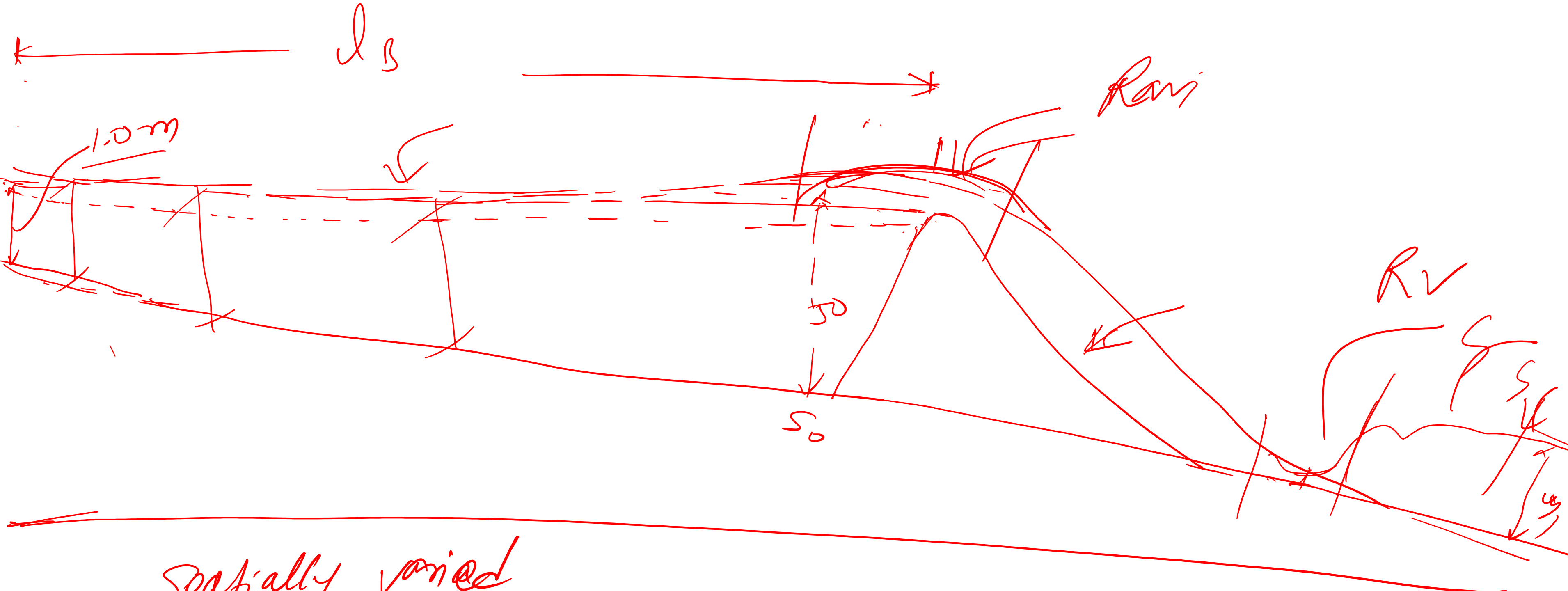
Gradually varied and Rapidly varied flow

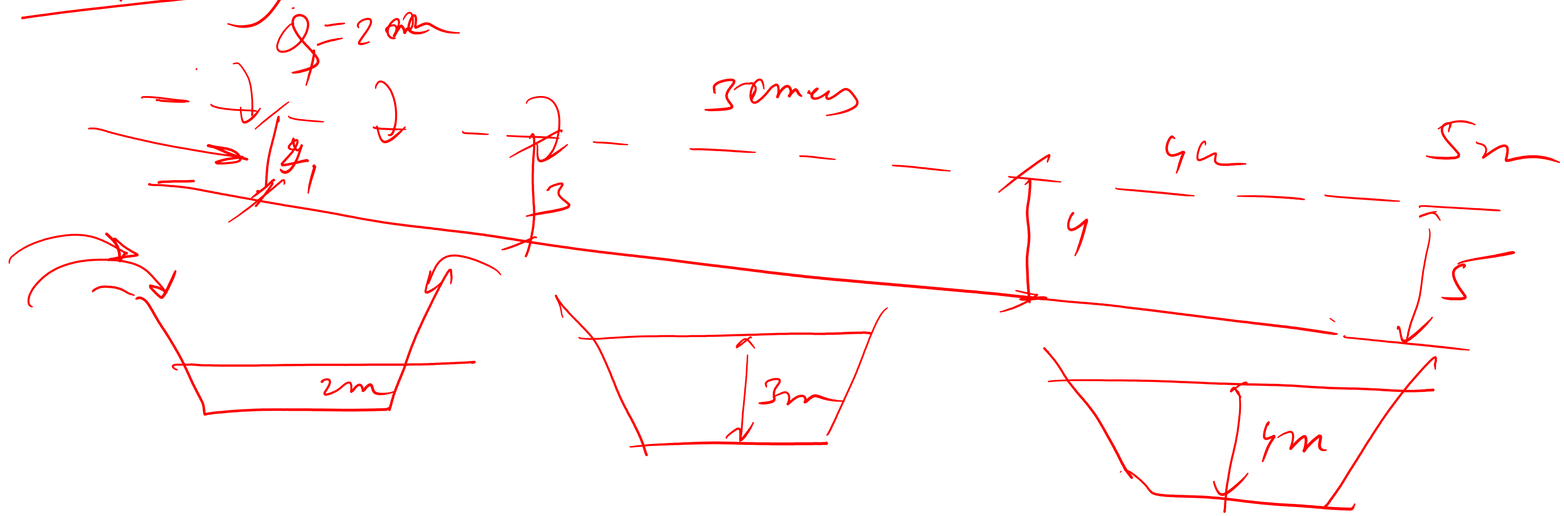
Gradually varied flow : A steady nonuniform flow in a prismatic channel where flow depth changes gradually along length of channel .

For example: Back water curve behind dam/weir





Spatially varied



Ex — side channel spillway

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Rapidly varied flow : In rapidly varied flow, change in water surface curvature is large within short length of channel.

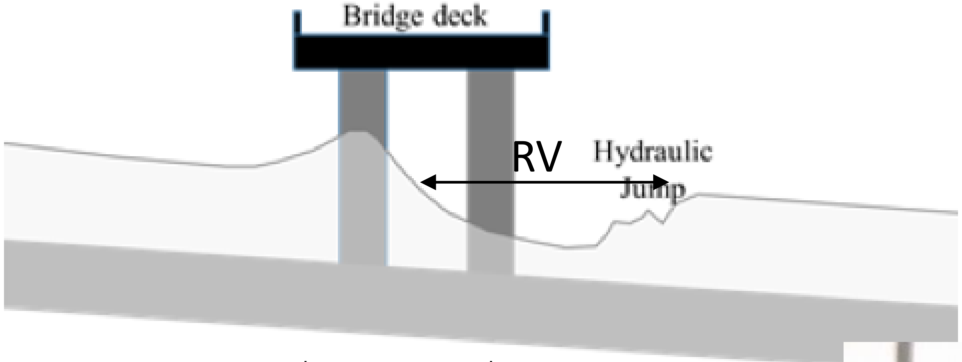
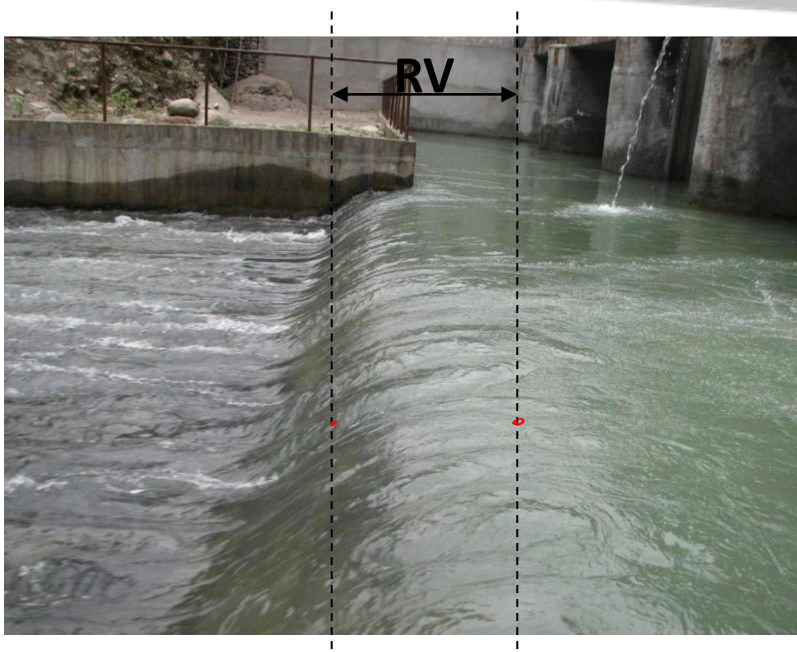
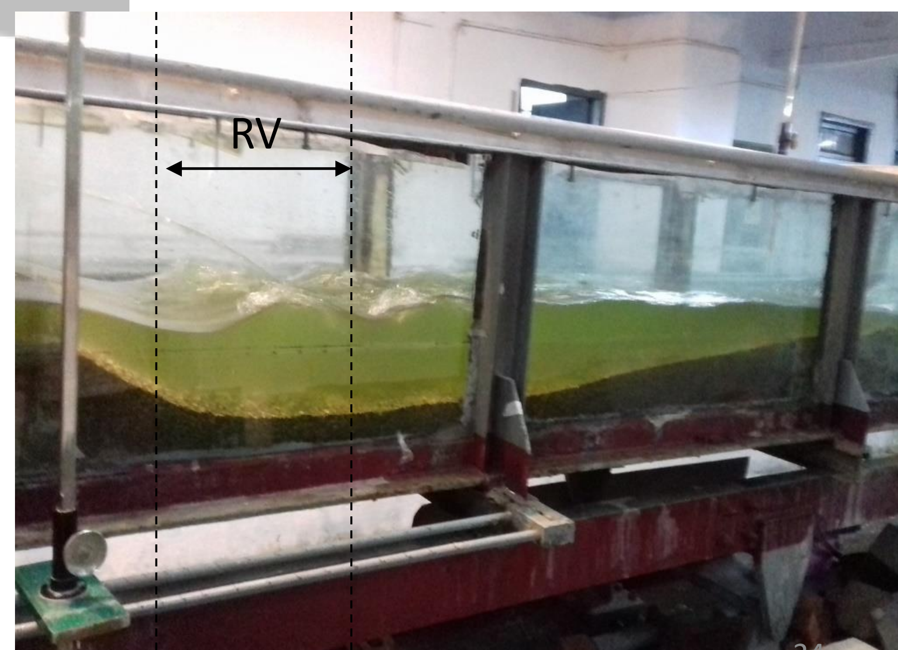
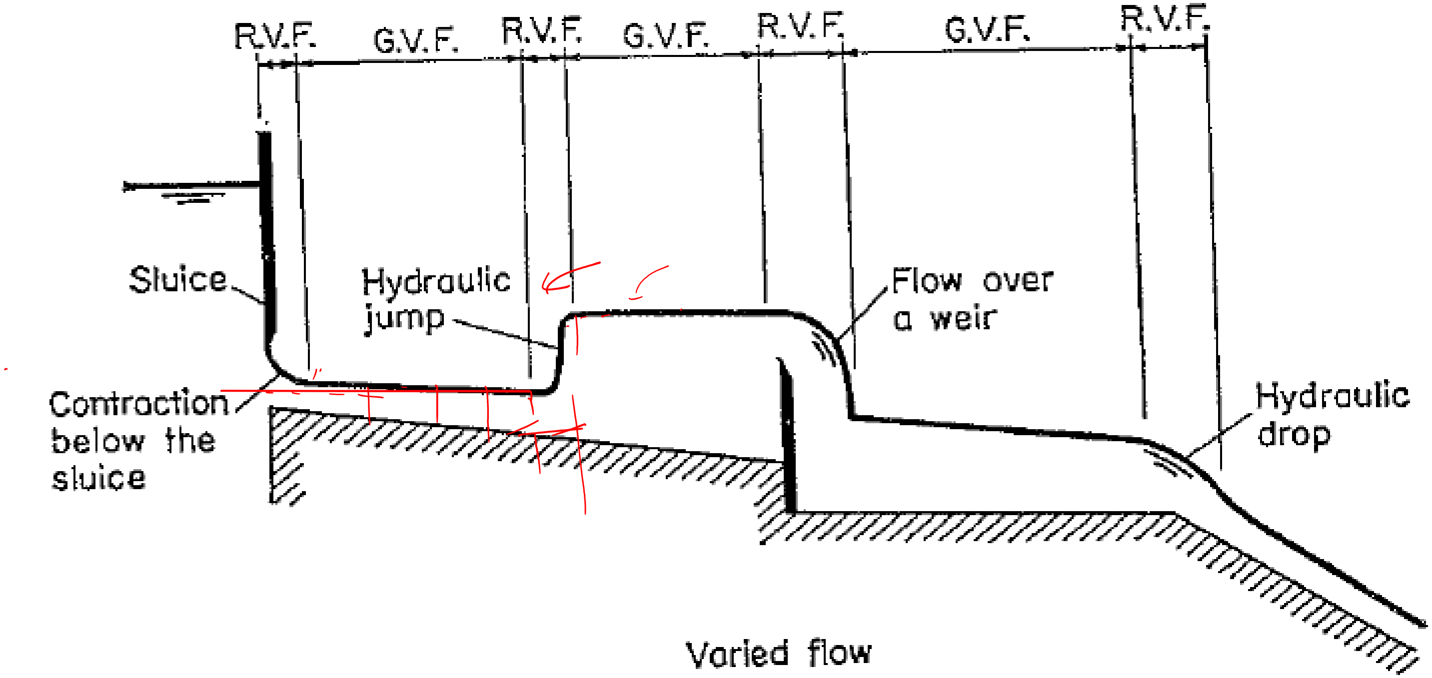


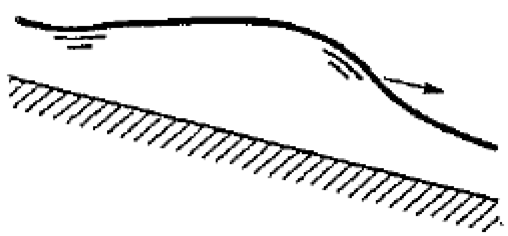
Fig.11: photographs of Rapidly varied flow at weir and in a experimental flume



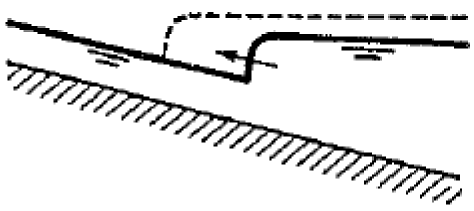




Varied flow



G.V.F. – Flood wave



R.V.F. – Bore

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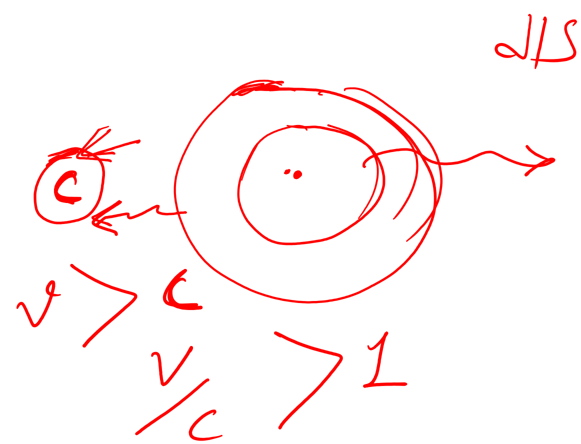
Subcritical flow : When Froude number of flow is less than one. Fr<1, For example flows in most of the rivers are subcritical flow

Supercritical flow : The flow having Froude number greater than one. Fr>1.

In order to examine whether the flow is subcritical or supercritical let us throw a stone in a flowing stream. If the ripple formed propagates in both upstream and downstream then the flow is subcritical. If the wave propagates only in downstream then the flow is supercritical.

Basic Equation for channel hydraulics

- Continuity equation
- Energy equation
- Momentum equation



$v = c$

$\frac{v}{c} = 1$

$\frac{v}{\sqrt{gy}} = 1$

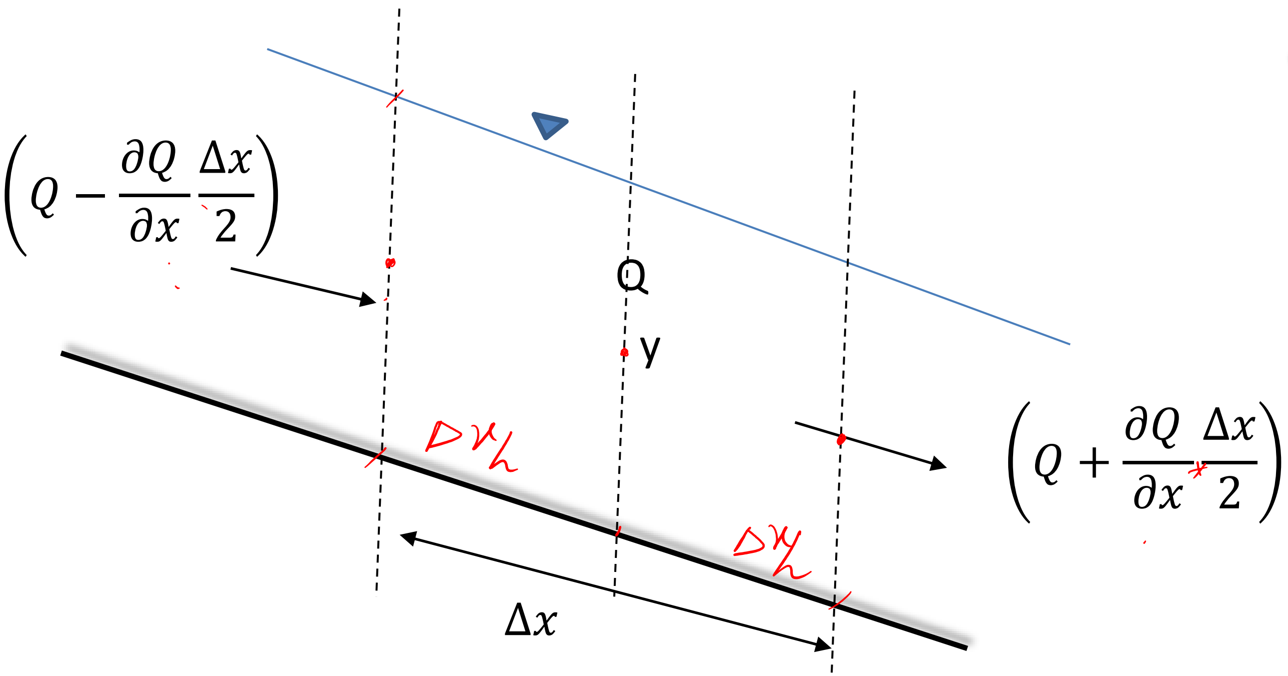
$v = \sqrt{gy}$ = critical flow

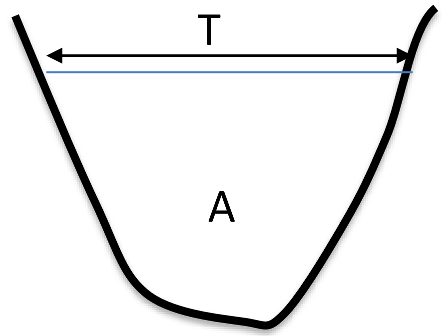
↓

V_c = critical velocity

$V_c = \sqrt{gy}$ → y_c = critical

Continuity equation





The net inflow in the control volume in time Δt,

$$\left[\left(Q - \frac{\partial Q}{\partial x}\frac{\Delta x}{2}\right) - \left(Q + \frac{\partial Q}{\partial x}\frac{\Delta x}{2}\right)\right]\Delta t = -\frac{\partial Q}{\partial x}\Delta x \Delta t \quad \text{..................(1)}$$

Also the increase in volume of the element in time Δt,

$$\frac{\partial}{\partial t}(A\Delta x)\,\Delta t \quad \text{.......(2)}$$

Contd:

Equating (1) and (2), and dividing by $\Delta x\ \Delta t$,

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t} = 0 \qquad \text{...........(3)}$$

But Q= AU, then (3) becomes,

$$\frac{\partial (AU)}{\partial x}+\frac{\partial A}{\partial t} = 0 \qquad \text{..........(4)}$$

Further, A=by for rectangular channel, then eqn.(4) becomes,

$$\frac{\partial (byU)}{\partial x}+\frac{\partial (by)}{\partial t} = 0$$

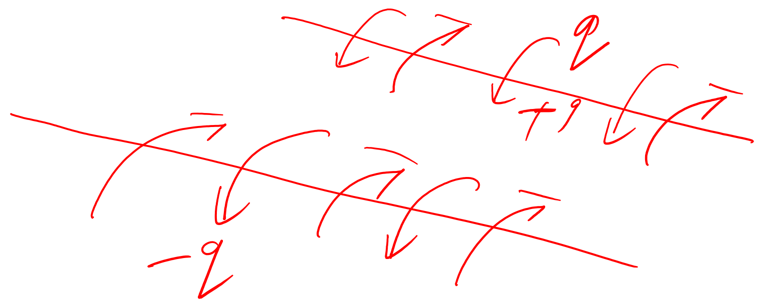
or, b $\left(\frac{\partial (yU)}{\partial x}+\frac{\partial y}{\partial t}\right)= 0$ or, $\frac{U\partial y}{\partial x} + \frac{y\partial U}{\partial x}+\frac{\partial y}{\partial t} =0$ ......(5)

For steady flow, $\frac{\partial A}{\partial t}$ =0 , then equation (4) becomes,

$$\frac{\partial (AU)}{\partial x} = \frac{\partial c}{\partial x}$$, thus $Q=A_1U_1=A_2U_2=A_3U_3=$……….(6)

If q be the lateral in flow or outflow from control volume, then

Eqn(4) becomes,



$$\frac{\partial (AU)}{\partial x}+\frac{\partial A}{\partial t} = \pm q$$

<u>Energy Equation,</u>

Let us consider non-curvilinear flow of channel section, where

pressure distribution can be assumed as hydrostatic.

Applying energy equation at section (1) and (2)

$$\frac{p_1}{\gamma} + z_1 + \frac{v1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v2^2}{2g} + hf$$

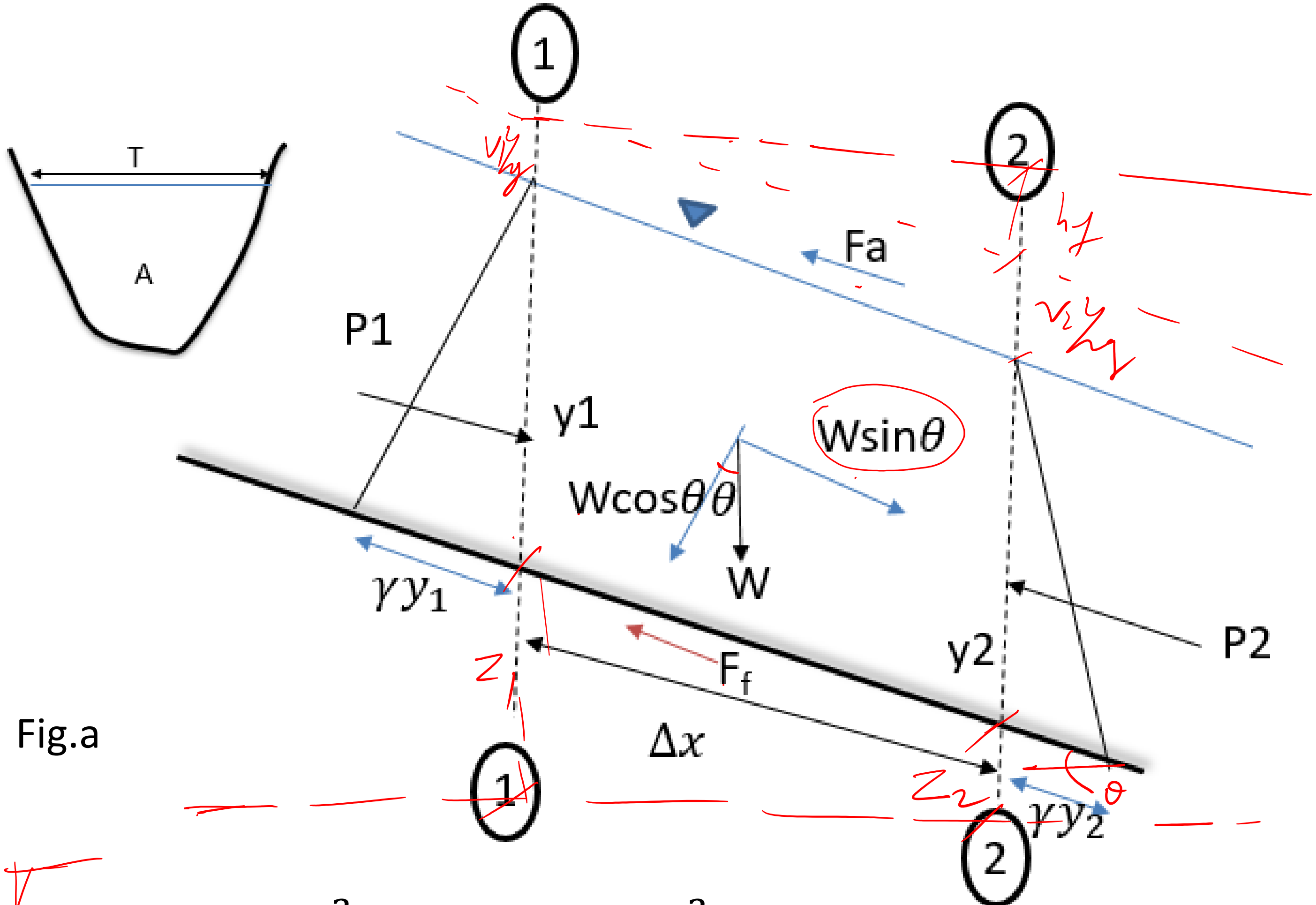


Fig.a

$$y_1 + z_1 + \frac{v1^2}{2g} = y_2 + z_2 + \frac{v2^2}{2g} + h_f$$, taking datum as channel bed, then

$$y_1 + \frac{v1^2}{2g} = y_2 + \frac{v2^2}{2g} + h_f \text{ , } \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$y_1 + \frac{v1^2}{2g}$ = constant …………(3)

Equation (3) is known as specific energy equation

Momentum equation

Considering figure (a), The net force along the x-direction must be equal to change in momentum flux

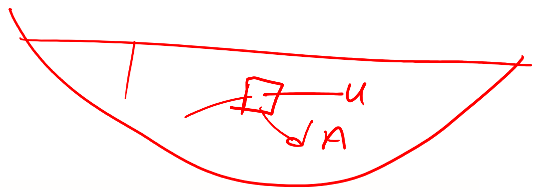
$\sum F_x$ = M_2-M_1

or, P1-P2-Ff-Fa+Wsinθ = $\rho Q(u_2 - u_1)$ ………(4)

Energy correction factor

In Ideal fluid , shear stress,τ at any section = 0 and thus flow velocity at each section of x-section becomes uniform.

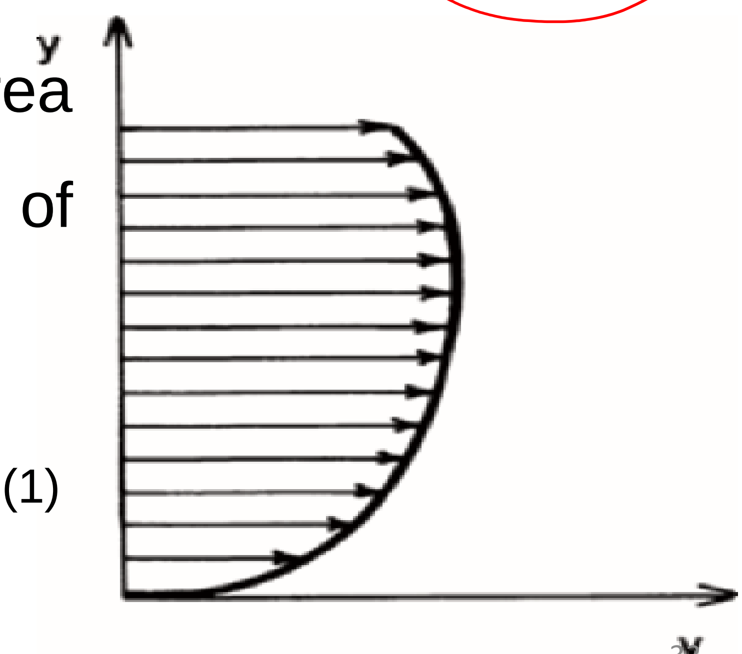
But for real fluid, it varies at each part of cross section of channel as shown in fig below . Flow is nonuniform across section. In order to consider this nonuniformity in the calculation of kinetic energy and momentum flux , velocity term has to be multiplied with some factor.



Consider a channel of x-section area A, in which u is the velocity of elementary area da, then the total

$\frac{1}{2}mv^2 = \rho\, dA\, v \times v^2/2g$

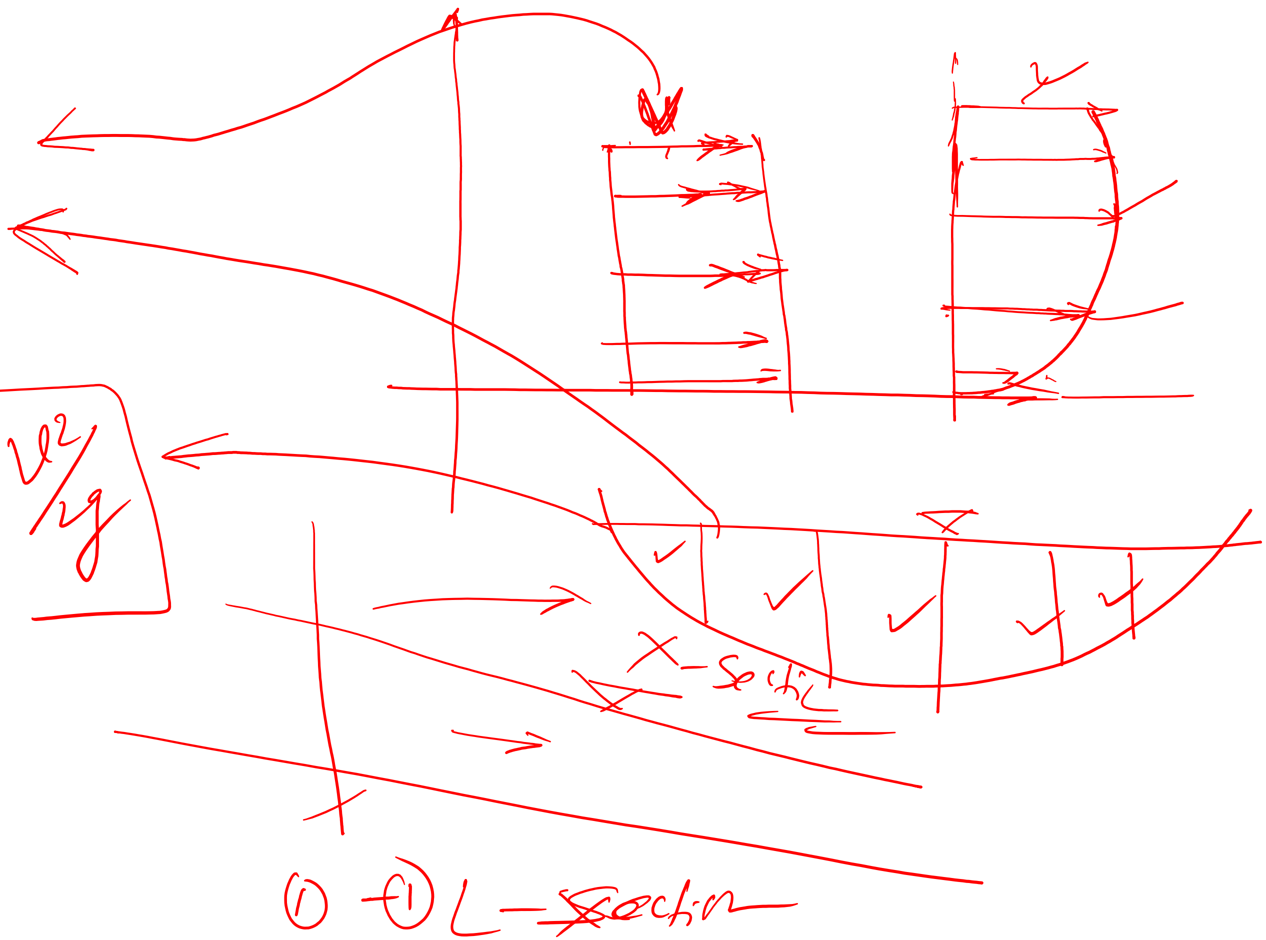
$$K.E. = \int_0^A \rho u dA \frac{u^2}{2} = \frac{1}{2}\int_0^A \rho u^3 dA \quad ....(1)$$



Energy correction factor (α)

$$E = y + \frac{V^2}{2g}$$

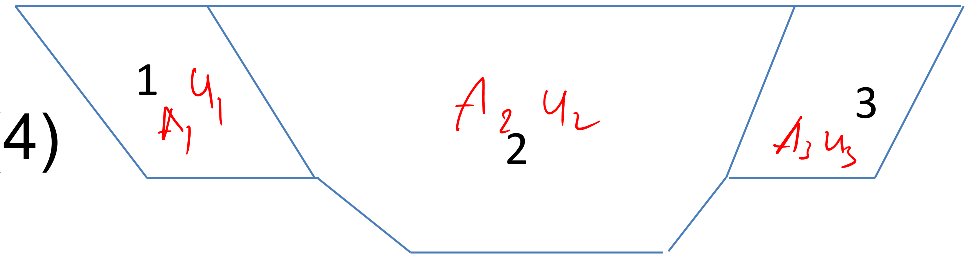
$$= y + \alpha \frac{V^2}{2g}$$



And KE using average velocity U= $m\frac{U^2}{2}$ = $(\rho UA \times U^2)/2$ = $\rho A\frac{U^3}{2}$ ……..(2)

Now α be KE correction factor , Then α= $(\frac{1}{2}\int_0^A \rho u\hat{}3dA)$/ $(\rho A\frac{U^3}{2})$

α= $\frac{1}{A}\int_0^A (\frac{u}{U})^3 dA$ ……………(4)



But A= by, then the above equation becomes

α= $\frac{1}{y}\int_0^y (\frac{u}{U})^3 dy$ …………………….(5)

For this typical river x-section,

α= $\frac{u_1^3 A1+u_2^3 A2+u_3^3 A3}{U^3(A1+A2+A3)}$ …………..(6)

Momentum Correction factor,

Similarly momentum flux of elementary area dA= $(\rho u dA)u$

Now, over area A = $\int_0^A \rho u^2 dA$ ……..(7)

And with uniform velocity, momentum flux over area A

$= \rho U^2 A$ ... ... (8)

If β be the momentum correction factor, then

$$\beta = \frac{\int_0^A \rho u^2 dA}{\rho U^2 A} = \frac{1}{A}\int_0^A (\frac{u}{U})^2 dA$$

A=by ,then

$$\beta = \frac{1}{y}\int_0^y (\frac{u}{U})^2 dy \quad \text{……..(8)}$$

For discrete area (above figure)

$\beta = \frac{u_1^2 A1 + u_2^2 A2 + u_3^2 A3}{U^2(A1+A2+A3)}$ ……(9). Thus our energy and momentum equation becomes

$y_1 + \alpha_1 \frac{v1^2}{2g}$ = constant …………(10)

$M_2 - M_1 = \rho Q(\beta_2 * u_2 - \beta_1 * u_1)$ …..(11)

Contd:

Velocity distribution, Shear stress distribution ,Pressure distribution

<u>Velocity distribution</u>

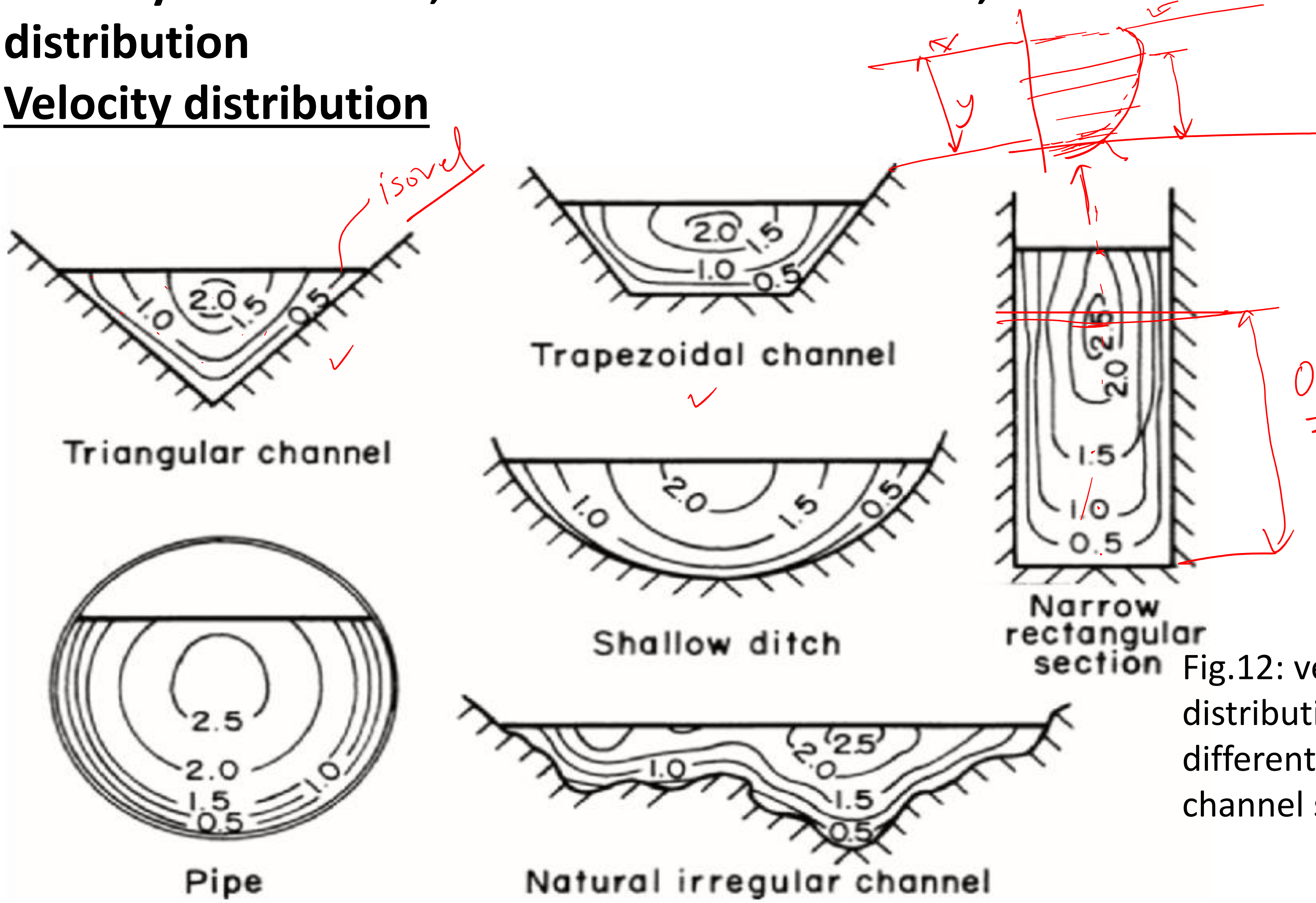
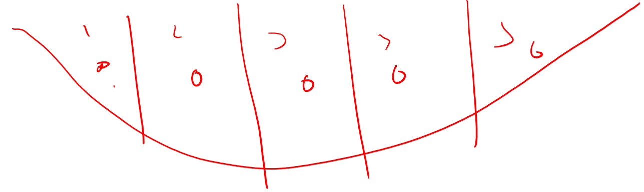


Fig.12: velocity distribution in different channel shape

Contd:



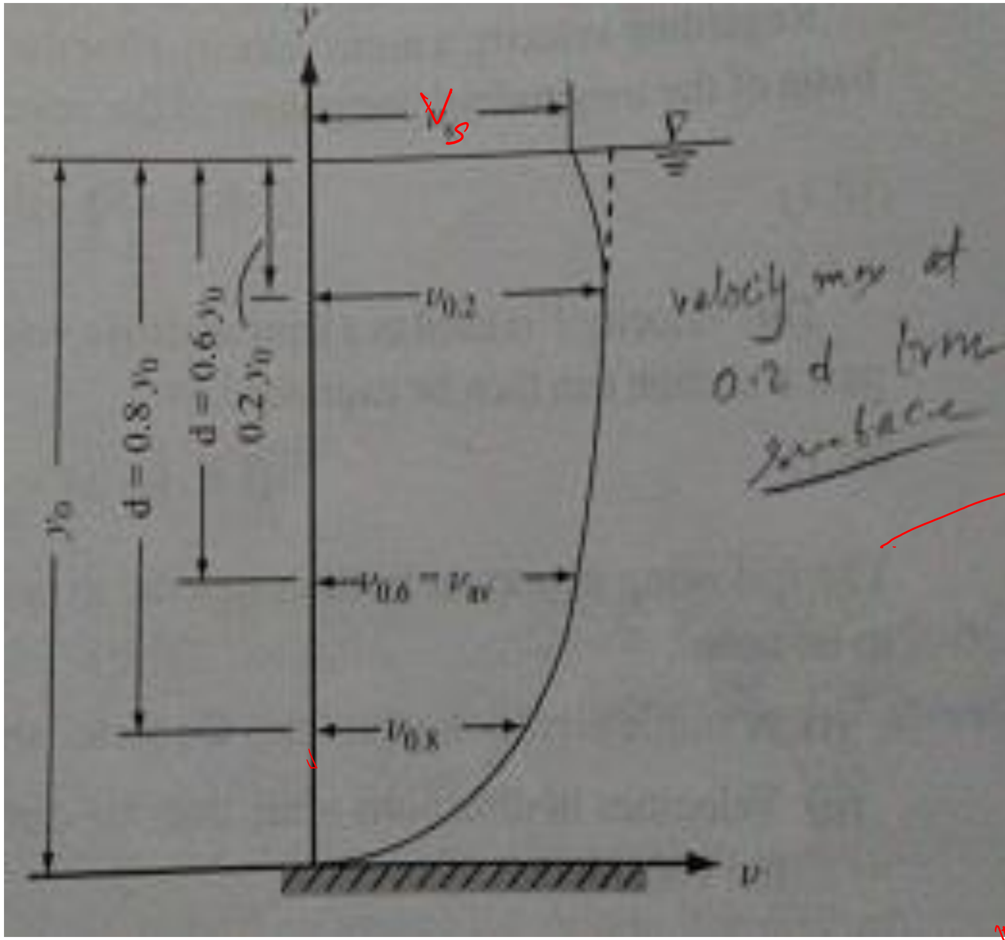
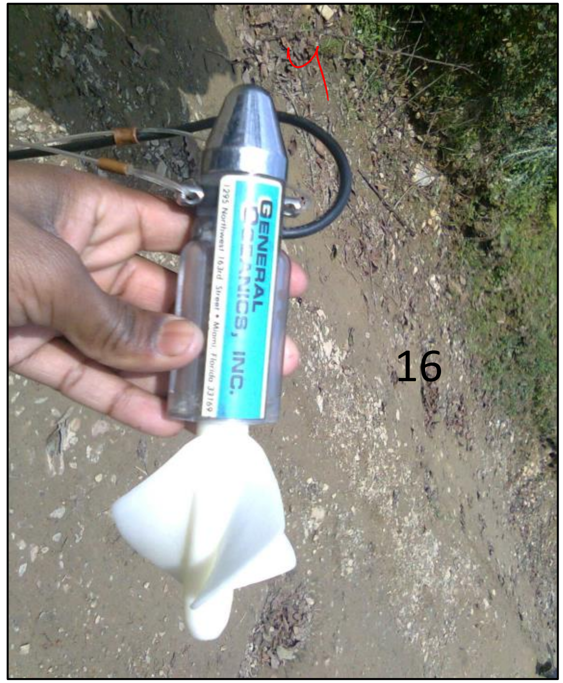
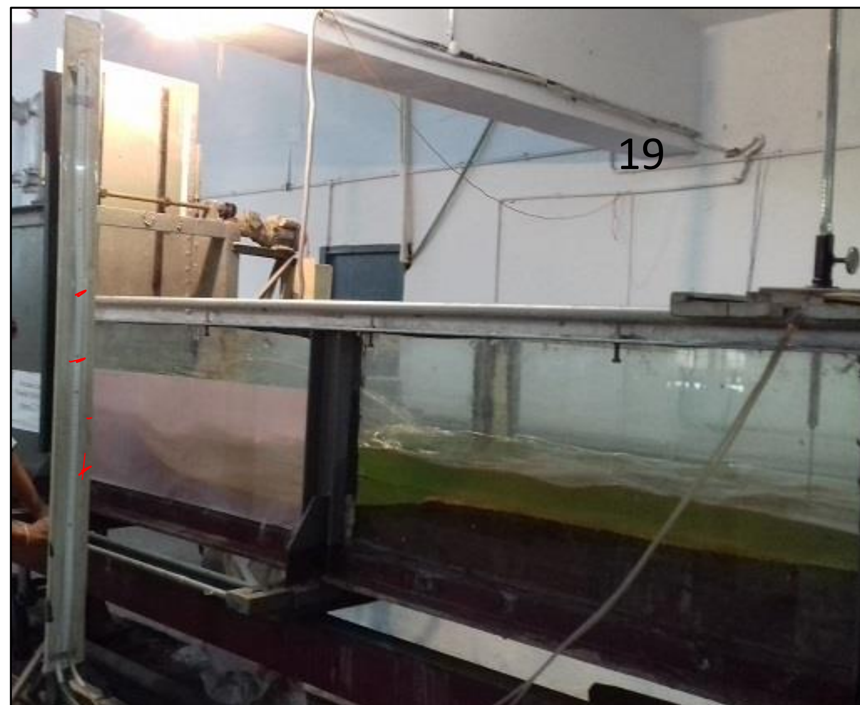


Fig. Typical velocity variation in open channel

Fig.13 & 14: Typical velocity profile in a channel and photographs 14 shows current meter used for velocity measurement in a stream, Panauti, Nepal

$$V_{av} = \frac{v_{0.2} + v_{0.8}}{2} \text{ also, } V_{av} = k\,V_s \ (k = 0.8\text{-}0.95)$$



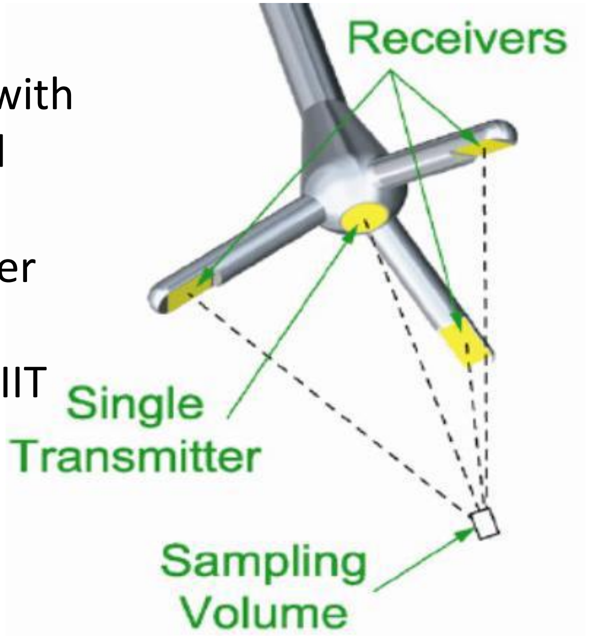


Δh

$V = \sqrt{2gh}$

$= \sqrt{2g}$

Velocity measurement with more advanced instruments, Acoustic Doppler Velocimeter at Hydraulics lab, IIT Roorkee



Ultrasonic flow meter for discharge measurement in pipe (Hydraulics lab, IIT Roorkee)

Contd:

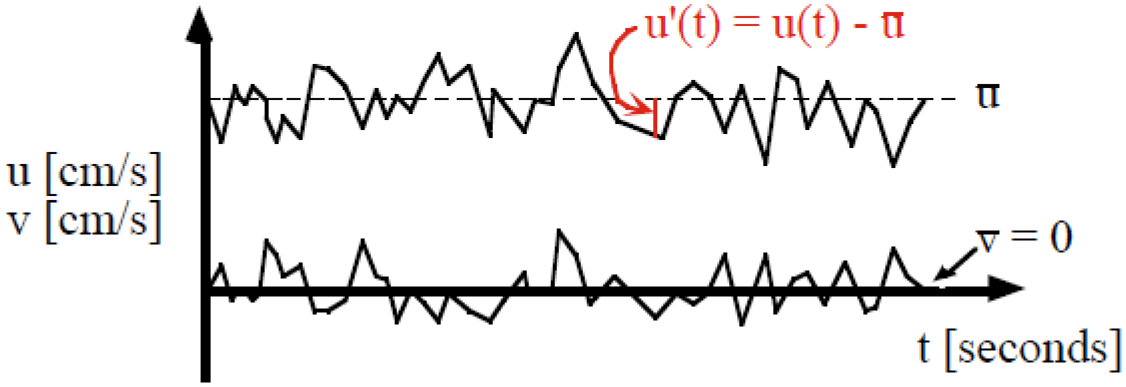


Fig. 7 : Velocity variation in quasi steady state

$$u(t) \quad = \quad \overline{u} \quad + \quad u'(t)$$

$$v(t) \quad = \quad \overline{v} \quad + \quad v'(t)$$

mean + turbulent fluctuation

Instantaneous velocity of flow along x, y and z direction

Contd:

Time average velocity at a point in a fluid flow is given as,

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt$$

Mean velocity: $\bar{u} = \int_t^{t+T} u(t)\,dt$ (continuous record) $= \frac{1}{N}\sum_1^N u_i$ (discrete, equi-spaced pts.)

Turbulent Fluctuation:

$u'(t) = u(t) - \bar{u}$: continuous record

$u'_i = u_i - \bar{u}$: discrete points

Turbulence Strength: $u_{rms} = \sqrt{\overline{u'(t)^2}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(u'_i)^2}$

Contd:

Velocity distribution in turbulent flow (Flow resistance equation)

Velocity distribution equation can be derived using relation of turbulent shear stress(Prandtl mixing length theory) and equation given by Nikuradse, i.e., l=ky where, k= von karman constant=0.4 (for details ,follow books)

τ = ρl^2 $\left(\frac{du}{dy}\right)^2$

Or, du = $\frac{1}{k}\frac{dy}{y}\sqrt{\frac{\tau_o}{\rho}}$, where, l=ky and $\sqrt{\frac{\tau_o}{\rho}}$ = u* shear velocity

Contd:

On integration both side, we get

u = $\frac{u*}{K}$ loge (y)+C ……………………………………………………….(1)

Applying boundary condition , at y' ,u=0 ; y' is a small

distance from boundary up to which u=0

Finally we get equation (5)

$\frac{u}{u_*}$ = 5.75 log (y/y') ……………………………………….(2)

Contd:

Hydrodynamically smooth and rough boundary

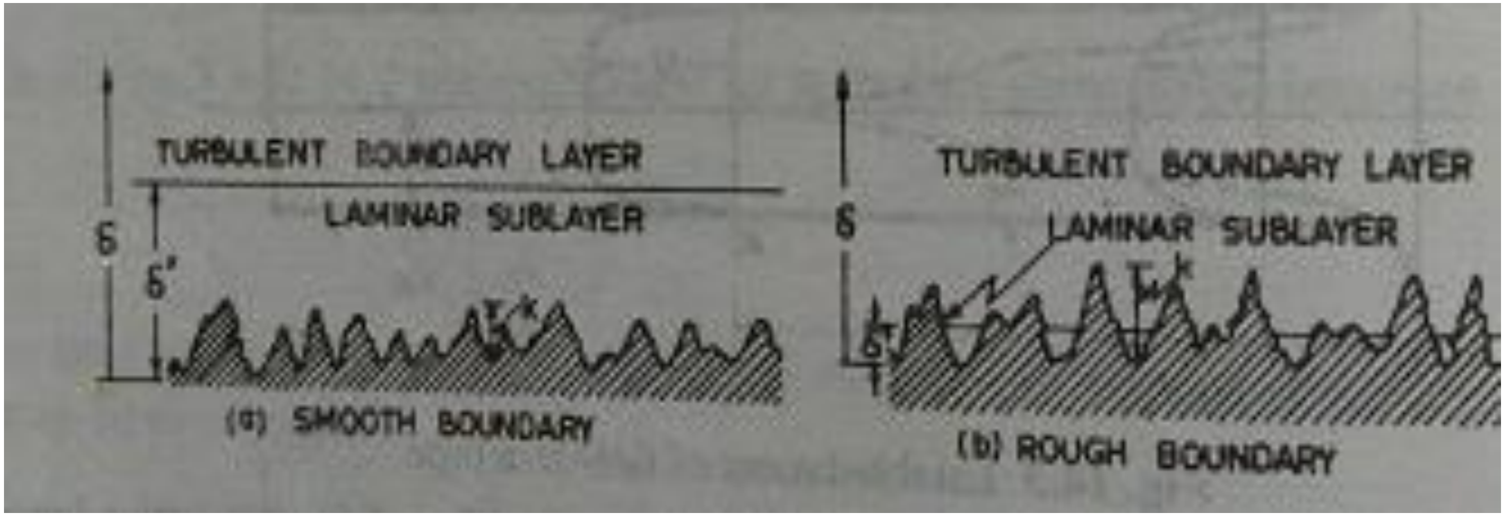


Fig.10:Schematic representation of smooth and rough boundary

For smooth surface ,$\frac{K_s}{\delta'} \leq 0.25$, and δ' = $\frac{11.6\gamma}{u_*}$;

For rough surface $\frac{K_s}{\delta'} \geq 6.0$; for smooth boundary, y' = $\delta'/107$

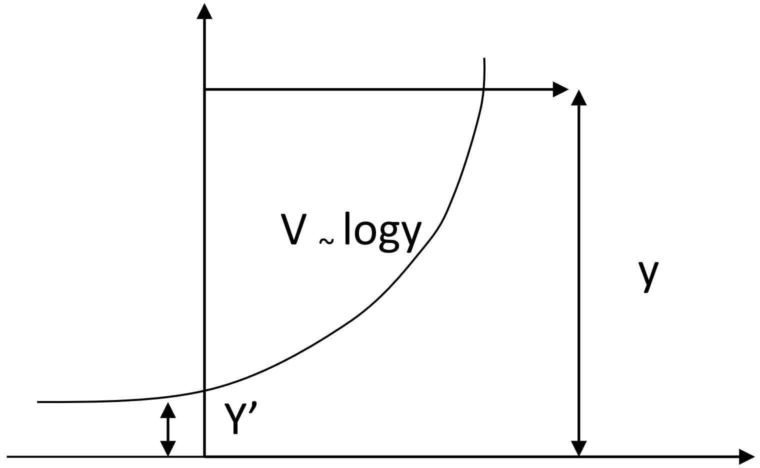
Where as for rough surface y'= Ks/30

Contd:

Now combining equation (4) with smooth and rough surface parameters, we get,

$$\frac{u}{u_*} = 5.75 \log \left(\frac{y\, u_*}{\gamma}\right) + 5.50 \qquad \text{............(5) (for smooth surface)}$$

$$\frac{u}{u_*} = 5.75 \log \left(\frac{y}{Ks}\right) + 8.50 \qquad \text{..........(6)(for rough surface)}$$



Contd:

Keulegan used equation 5 and 6 ,for the derivation of flow resistance equation in terms of mean velocity for smooth and rough channels

$$\frac{U}{u_*} = 5.75 \log \left(\frac{R\, u_*}{\gamma}\right) + 3.25 \quad \text{............(7) (for smooth channels)}$$

$$\frac{U}{u_*} = 5.75 \log \left(\frac{R}{Ks}\right) + 6.25 \quad \text{..........(8)(for rough channels)}$$

For boundary in transition surface, above equation can't be applied. So, Einstein and Barbarossa has given a resistance equation for transition surface,

$$\frac{U}{u_*} = 5.75 \log \left(\frac{12.27Rx}{Ks}\right) \quad \text{.......(9) for transition boundary, where } x = f\left(\frac{k_s}{\delta'}\right)$$

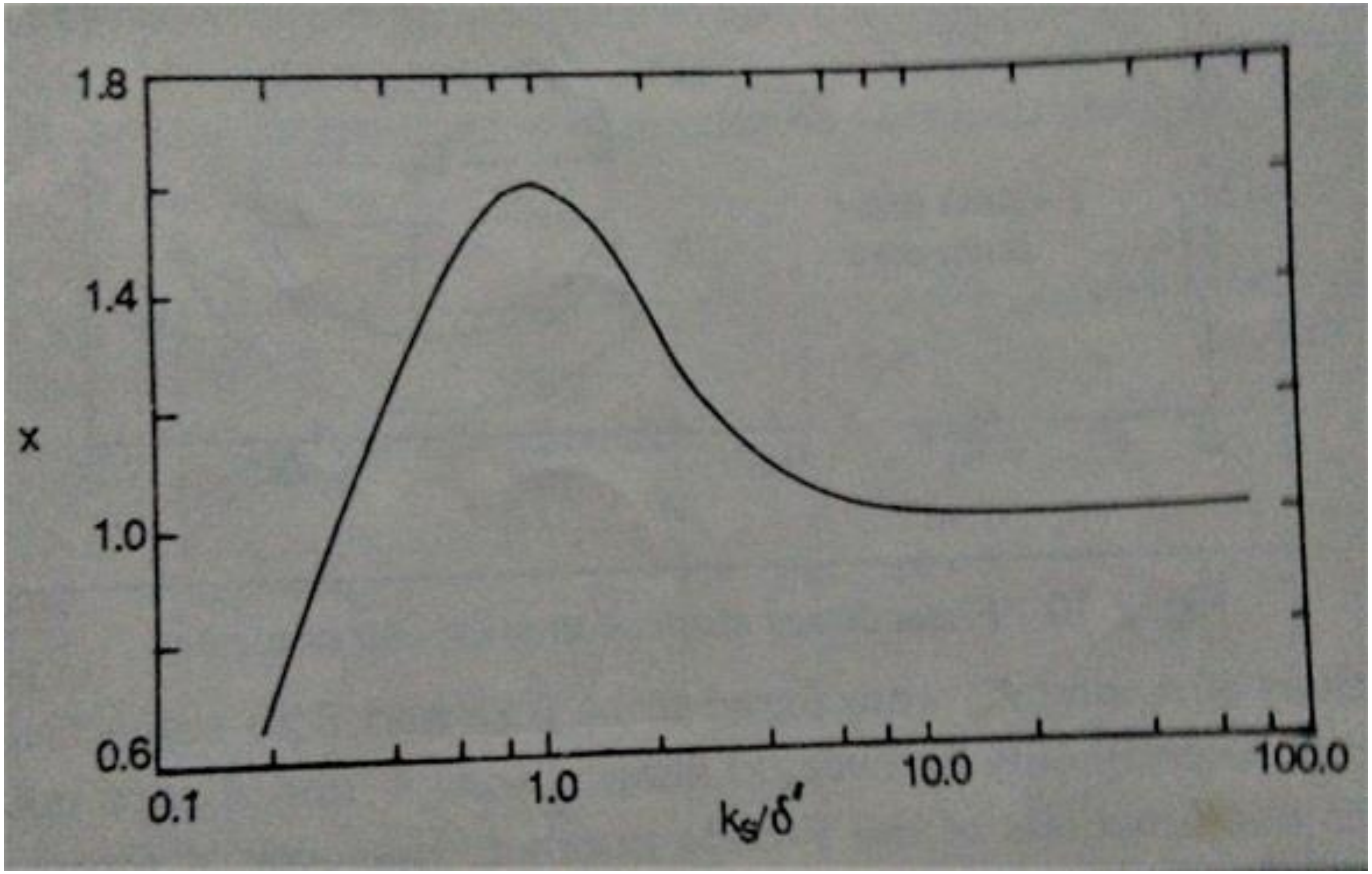
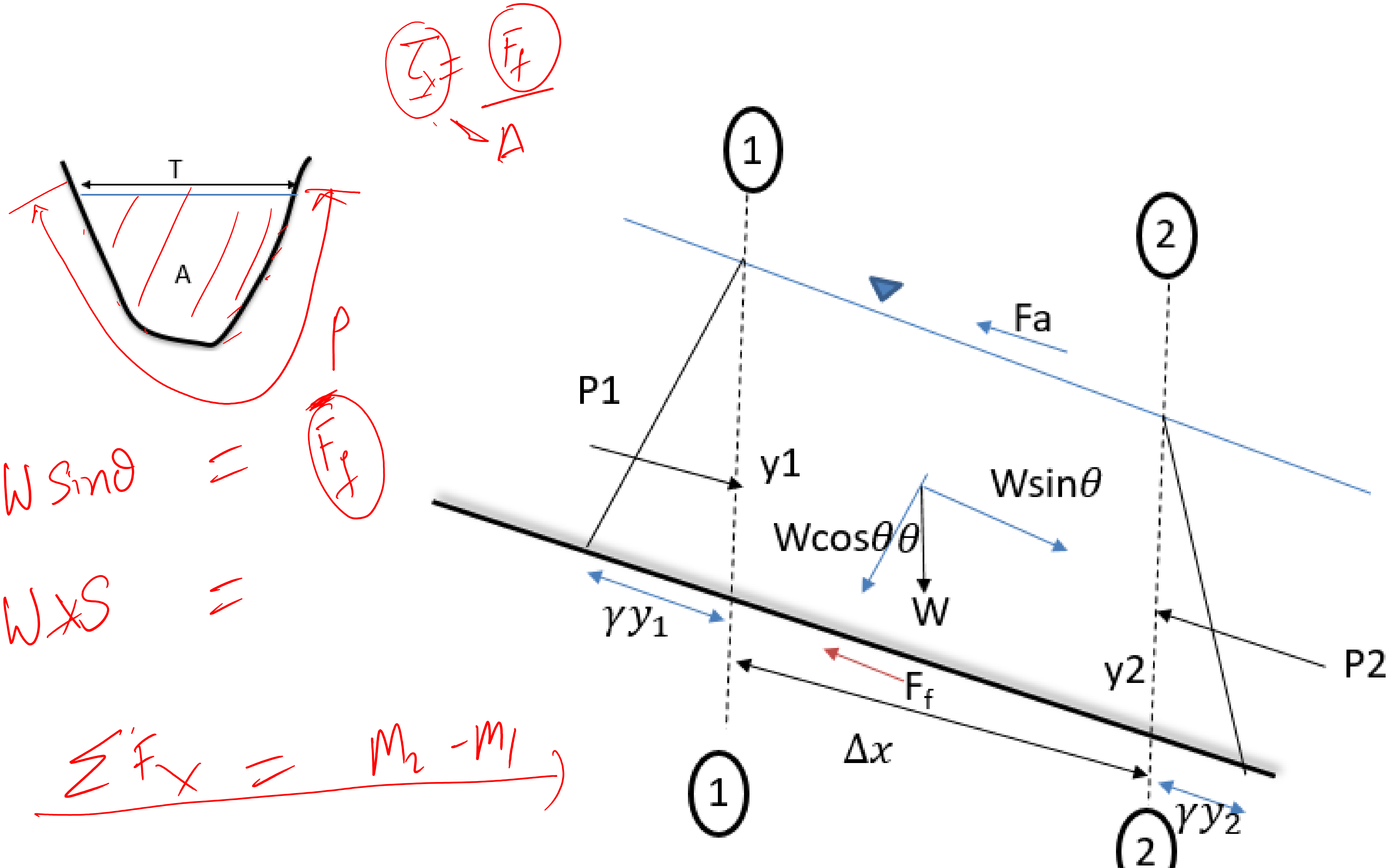


Fig.Correction factor for viscous effects in logarithmic formula given by Einstein

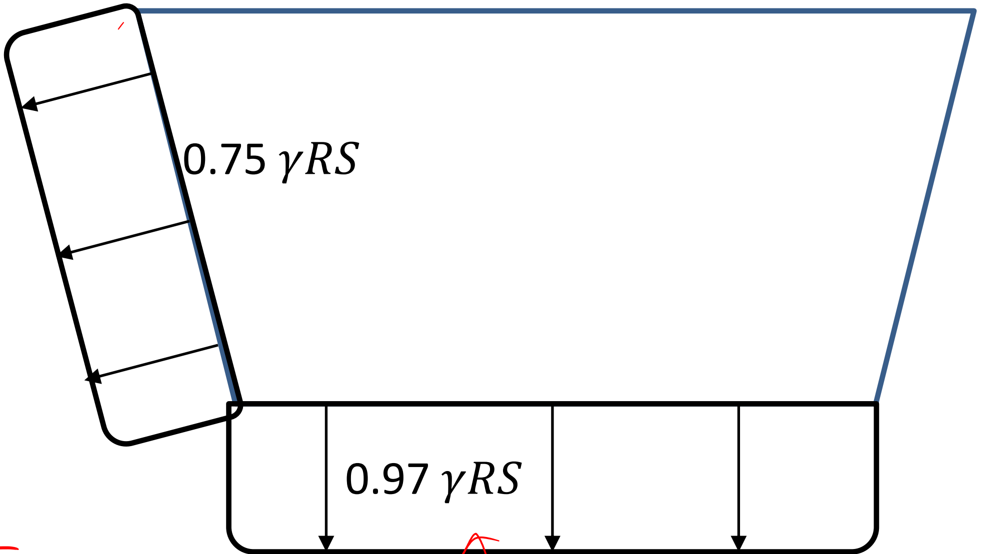
Expression for shear stress at channel boundary in Uniform flow

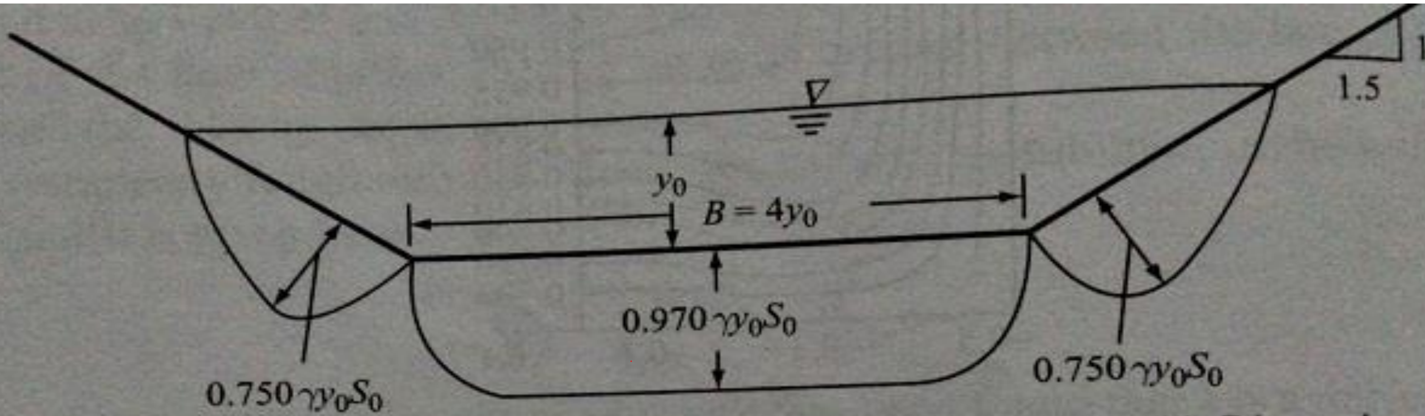


P1-P2+Wsinθ – Ff-Fa = $\rho Q(u_2 - u_1)$ but in Uniform flow P1=P2 and u1=u2
And neglecting Fa and sinθ ~ θ = S, slope of channel bed
then , Wsinθ – Ff=0, or,

WS=τA

τ = WS/A or, $\tau = \frac{\gamma ALS}{PL} = \gamma RS$





ig. 3.6 *Variation of boundary shear stress in a trapezoidal channel with $B/y_o = 4$ and $m = 1.5$*

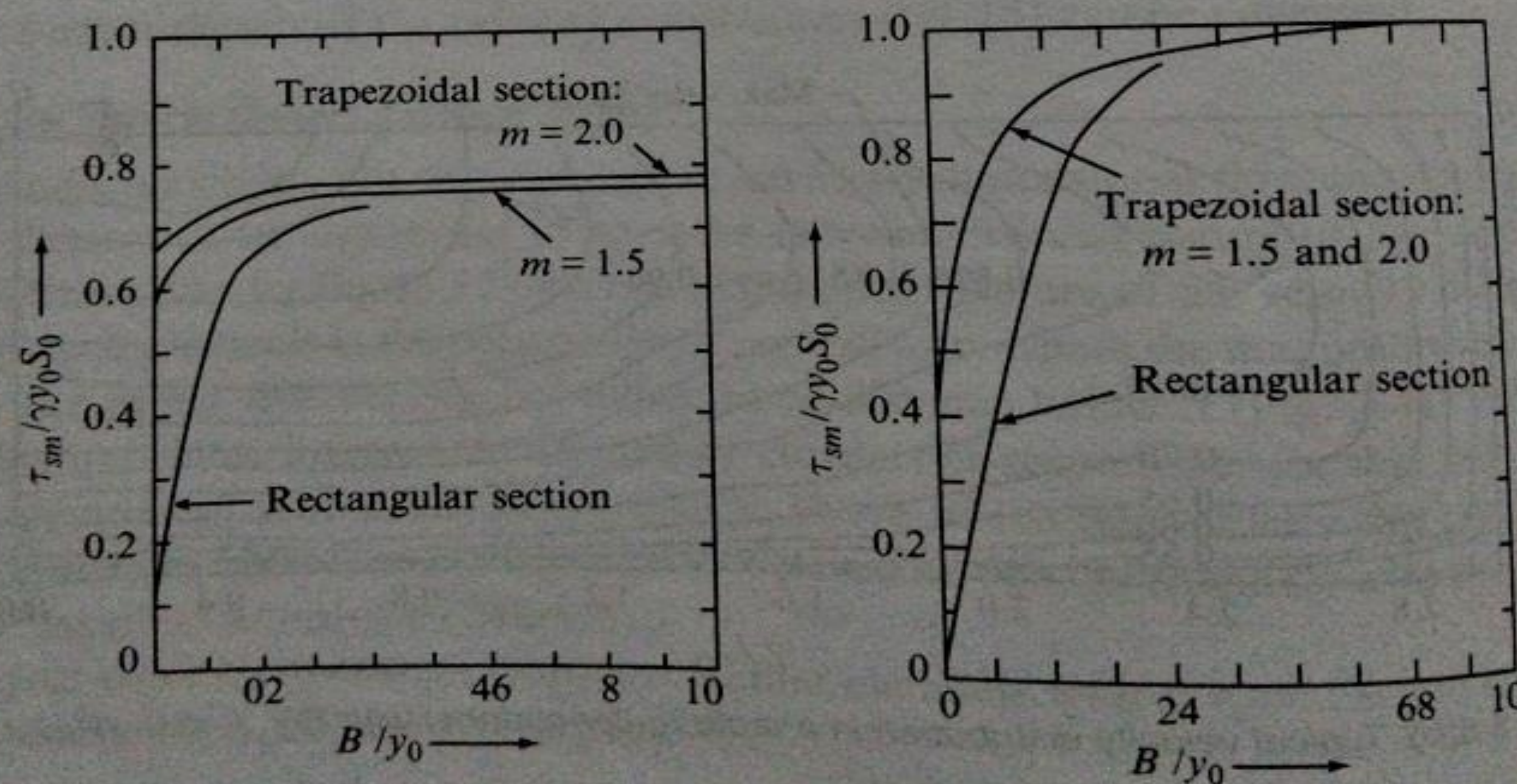
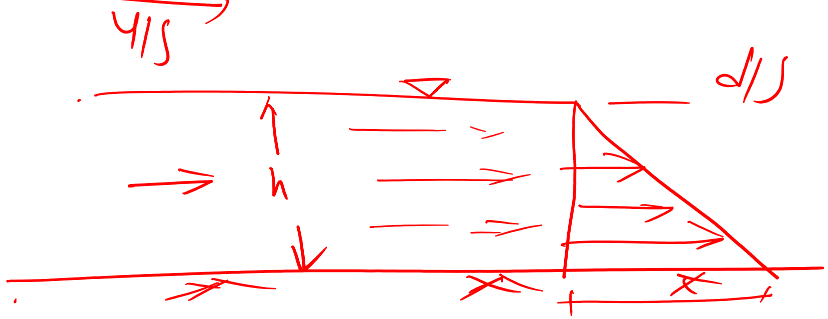


Fig. 3.7 *Variation of maximum shear stress on bed and sides of smooth channels*

Pressure distribution in open channel

Straight channel



$P = \rho g h$

$$a = a_s + a_n$$

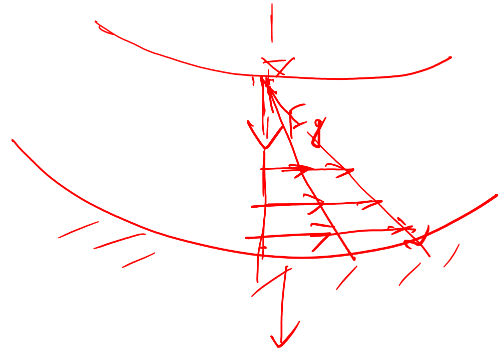
$$= \left(v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right) + \frac{v^2}{r}$$

Uniform

$$\frac{\partial v}{\partial s} = 0 \quad \& \quad \frac{\partial v}{\partial t} = 0$$

$$a_n = v^2/r$$

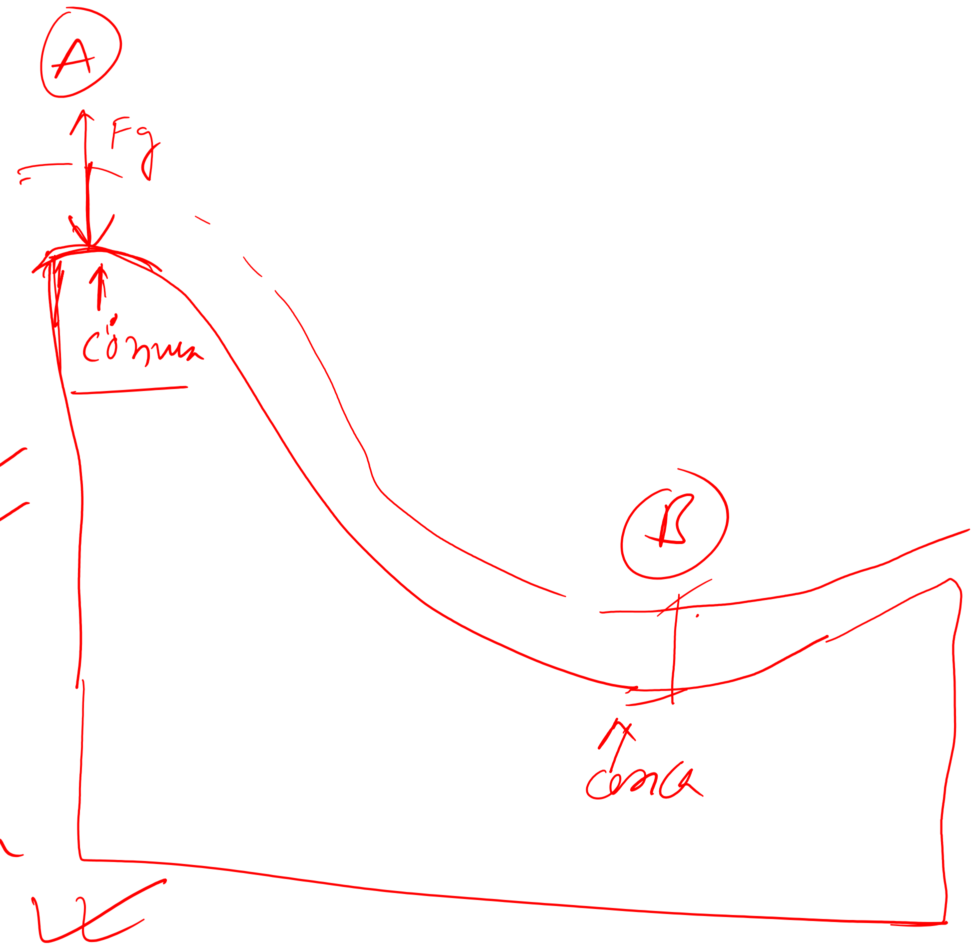
concave



$$P = \rho g h + \rho\left(\frac{v^2}{r}\right)h$$

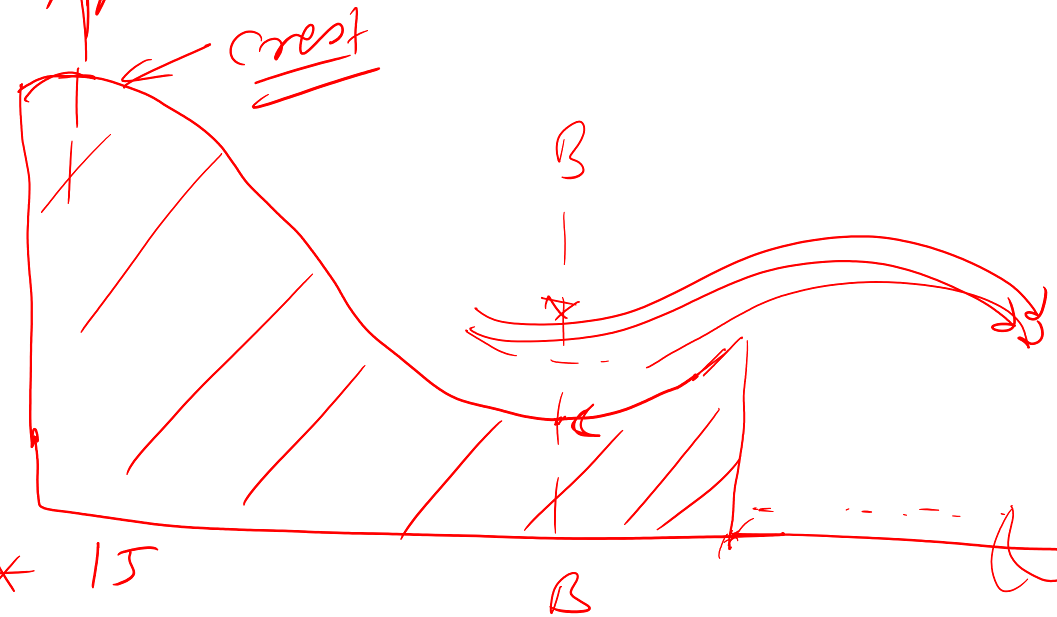
convex boundary

$$P = \rho g h - \rho\left(\frac{v^2}{r}\right)h$$



Q

A spillway flip bucket has a radius of 20 m. If the flow velocity at section B-B is 20 m/s and the flow depth is 15 m. compute the pressure intensity at point C.



$$P = \rho g h + \rho \left(\frac{v^2}{r}\right) h$$

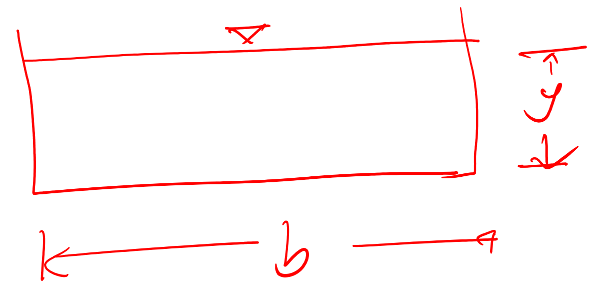
$$= 1000 \times 9.81 \times 15 + 1000 \times \left(\frac{(20)^2}{20}\right) \times 15$$

$$= \square \ N/m^2$$

Flow resistance eqn.

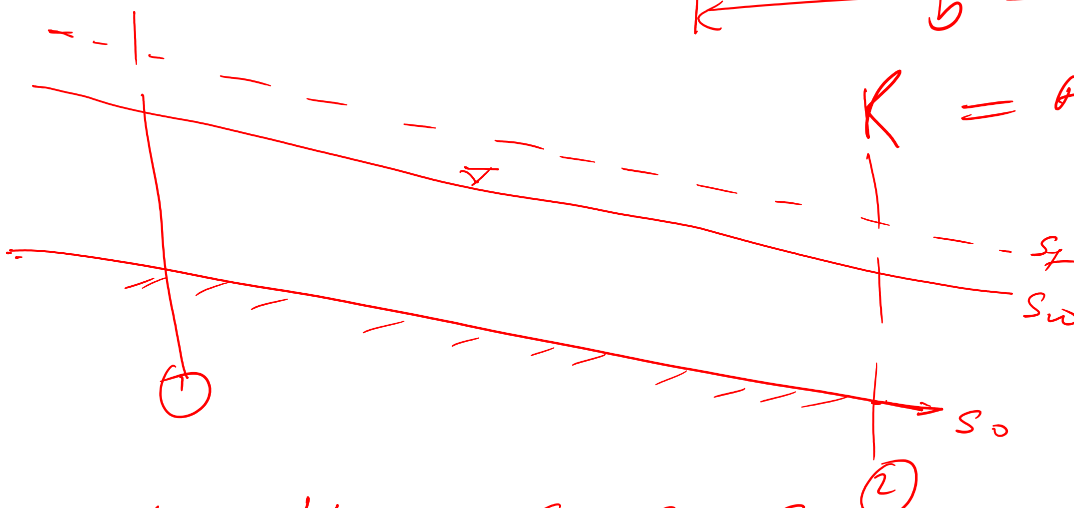
1. Chezy's formula
2. Manning's formula
3. Darcy weisbach formula

→ Uniform flow



① $V = C\sqrt{RS}$

$R = A/P = \frac{by}{b+2y}$



at uniform flow $S_0 = S_w = S_f$

C = Chezy's constant

Some important flow resistance formula : Chezy, Manning's formula

The shear stress at boundary of channel $\tau_o = \gamma RS$ ..(1) and also,

Shear stress exerted at boundary can be written in terms of surface drag force, Drag force = $C_d \frac{A\rho u^2}{2}$, thus shear stress τ = $\frac{C_d \frac{A\rho u^2}{2}}{A}$ = $C_d \frac{\rho u^2}{2}$ …….(2), where A is wetted perimeter area of channel

Equating 1 and 2

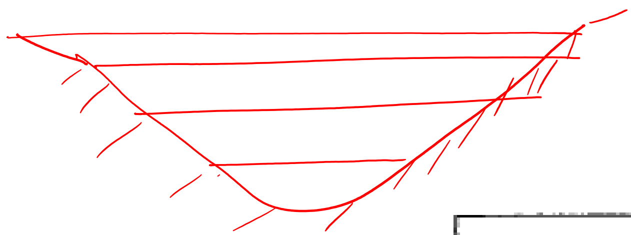
We get $u = C\sqrt{RS}$

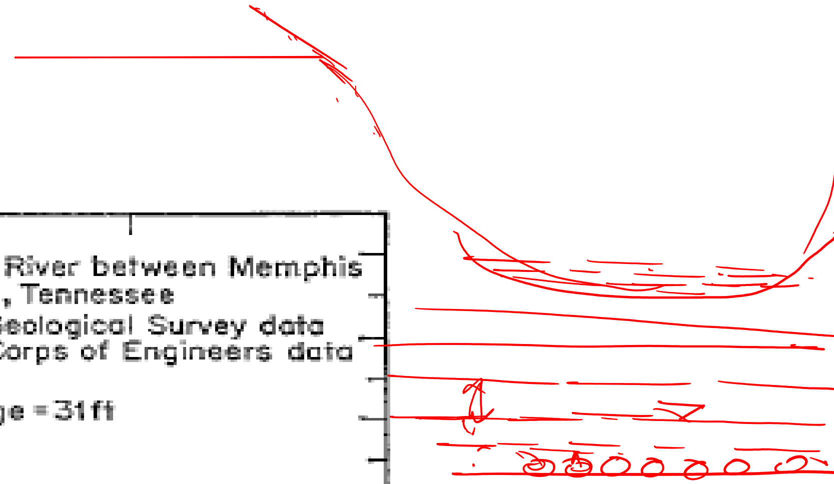
Chezy's coefficient can be estimated using Kutter's formula (follow book)

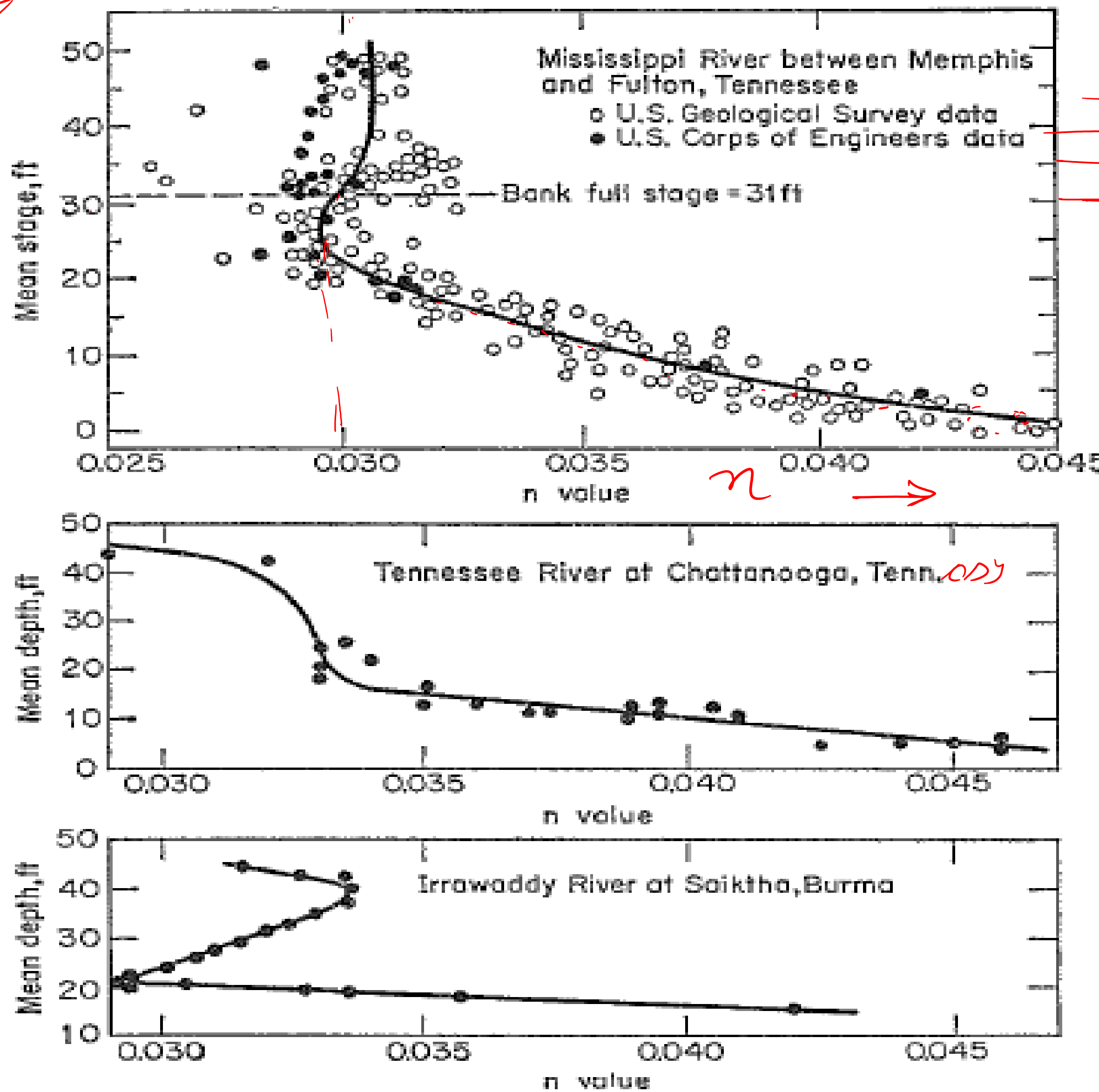
Irrigation

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}}$$

S = channel bed






Mississippi River between Memphis and Fulton, Tennessee
o U.S. Geological Survey data
• U.S. Corps of Engineers data
Bank full stage = 31 ft
Mean stage, ft
n value
Tennessee River at Chattanooga, Tenn.
Mean depth, ft
n value
Irrawaddy River at Saiktha, Burma
Mean depth, ft
n value

Manning's Formula

C=$\frac{R^{1/6}}{n}$ ..... Derive Manning's formula

- Relationship between Darcy-weisbach friction factor, Chezy eqn and Manning's equation

Put C in Chezy equation

U= $\frac{R^{1/6}}{n}$ $*\sqrt{RS}$

Or , U = $\frac{1}{n}R^{2/3}S^{1/2}$

Similarly from, $\frac{U}{U_*}$= $\sqrt{\frac{8}{f}}$ or, U= $U_*\sqrt{\frac{8}{f}}$ = $\sqrt{\frac{8g}{f}}\sqrt{RS}$

Thus, U= $\sqrt{\frac{8g}{f}}\sqrt{RS}$ = $\frac{1}{n}R^{2/3}S^{1/2}$= C $\sqrt{RS}$ (relations to be remembered)

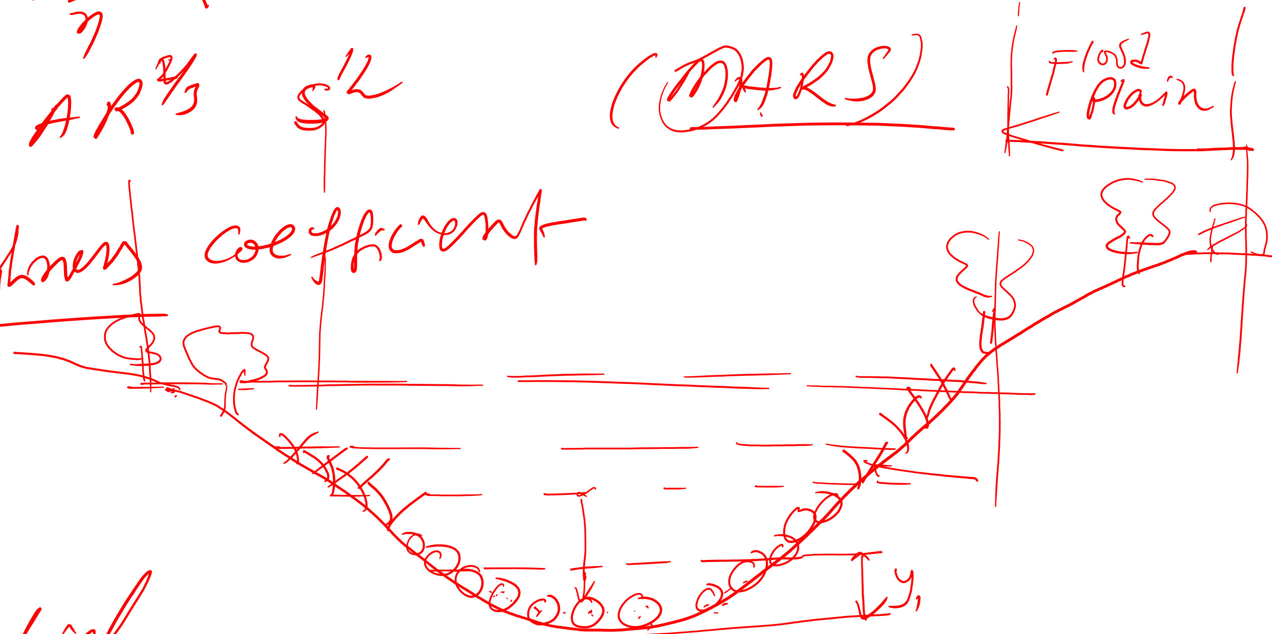
$V = \frac{1}{n} R^{2/3} S^{1/2}$

$Q = AV = A \times \frac{1}{n} R^{2/3} S^{1/2}$

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$ (MARS)

n = manning's roughness coefficient

factors affecting n



natural channel

(i) flow depth

(ii) bed & bank material

(iii) Sediment content of flowing water

(iv) Trees, slope

Slope ✓

will be low for mild slope

$F_{xf} = \quad n =$ will be low for mild slope

$$F = C_d A \rho u^2/2$$

$$F \propto u^2$$

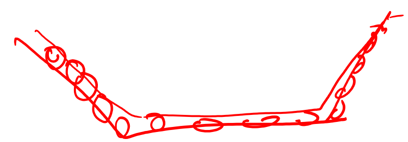
we must prismatic channel

①

$n = 0.011$

~~0.013~~

$n = 0.018$



$n = 0.015$

0.026

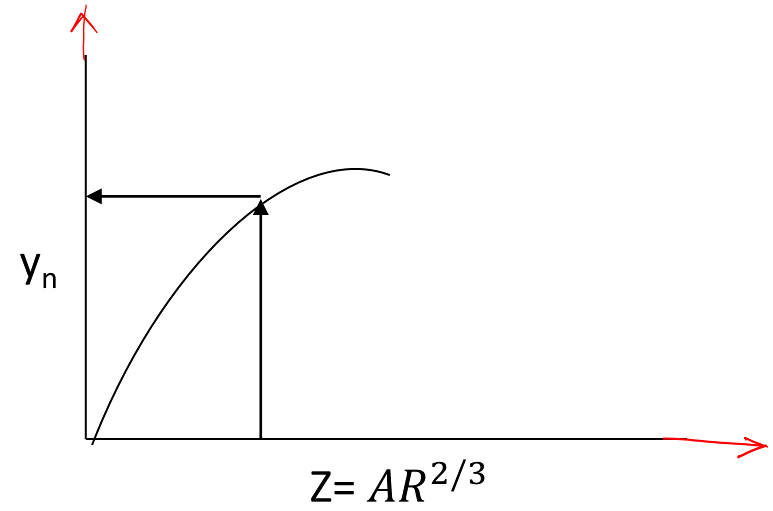
Variation of hydraulic radius with depth of flow

R Vs y

$R = A/P$

- Circular
- Rectangular
- Triangular
- Trapezoidal

- Factors affecting M anning's n
- Conveyance
- section factor of channel



Q= AU = $\frac{1}{n}AR^{2/3}S^{1/2}$

K= Conveyance (channel carrying capacity) = $\frac{1}{n}AR^{2/3}$

Section factor, Z= $AR^{2/3}$,

Thus Q= (Z $S^{1/2}$)/n

Z= Qn/ $S^{1/2}$

Section factor is a unique function of flow depth. i.e., Z= f(y) , keeping width, discharge and slope of channel constant. There is only one value of depth of flow of channel from above figure and such depth is known as normal depth of flow.

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} A R^{2} S^{1/2}$$

→ X-sectional Propes

$$Z = A R^{2/3}$$

$$K = \frac{1}{n} A R^{2/3}$$

$$n = \frac{d_{50}^{1/6}}{21.1}$$, Strickler formula

$$n = \frac{d_{90}^{1/6}}{26}$$,Modified by Meyer. et al.

Horton’s Method of Equivalent roughness estimation

$$n = \frac{\left(\sum n_i^{3/2} P_i\right)^{2/3}}{P^{\frac{2}{3}}}$$

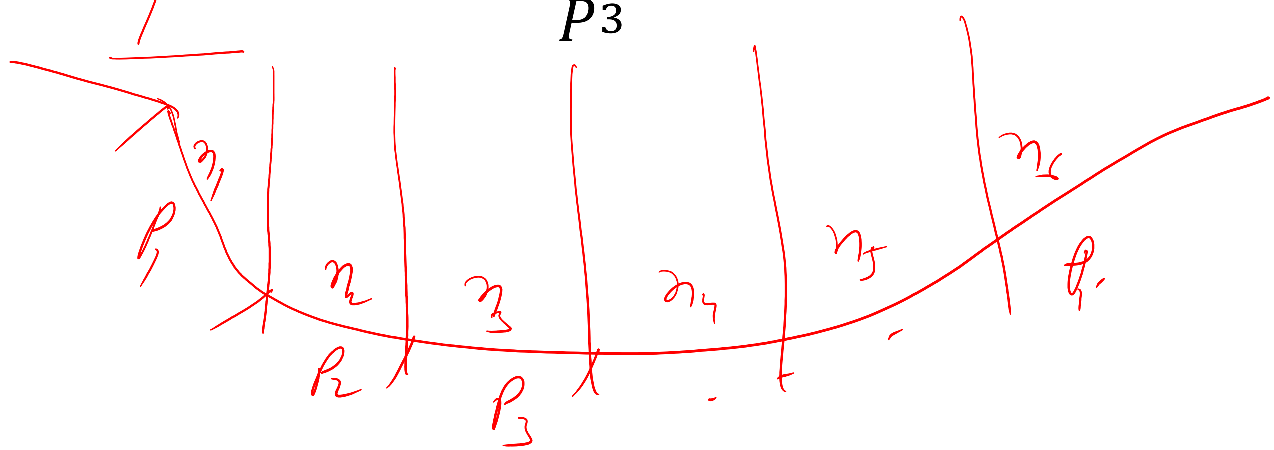


Table 4-1. Typical values* of Manning n

Material	n
Metals	
Steel	0.012
Cast iron	0.013
Corrugated metal	0.025
Non-metals	
Lucite	0.009
Glass	0.010
Cement	0.011
Concrete	0.013
Wood	0.012
Clay	0.013
Brickwork	0.013
Gunite	0.019
Masonary	0.025
Rock cuts	0.035
Natural streams	
Clean and straight	0.030
Bottom: gravel, cobbles and boulders	0.040
Bottom: cobbles with large boulders	0.050

*Compiled from tables presented by Chow [1959].

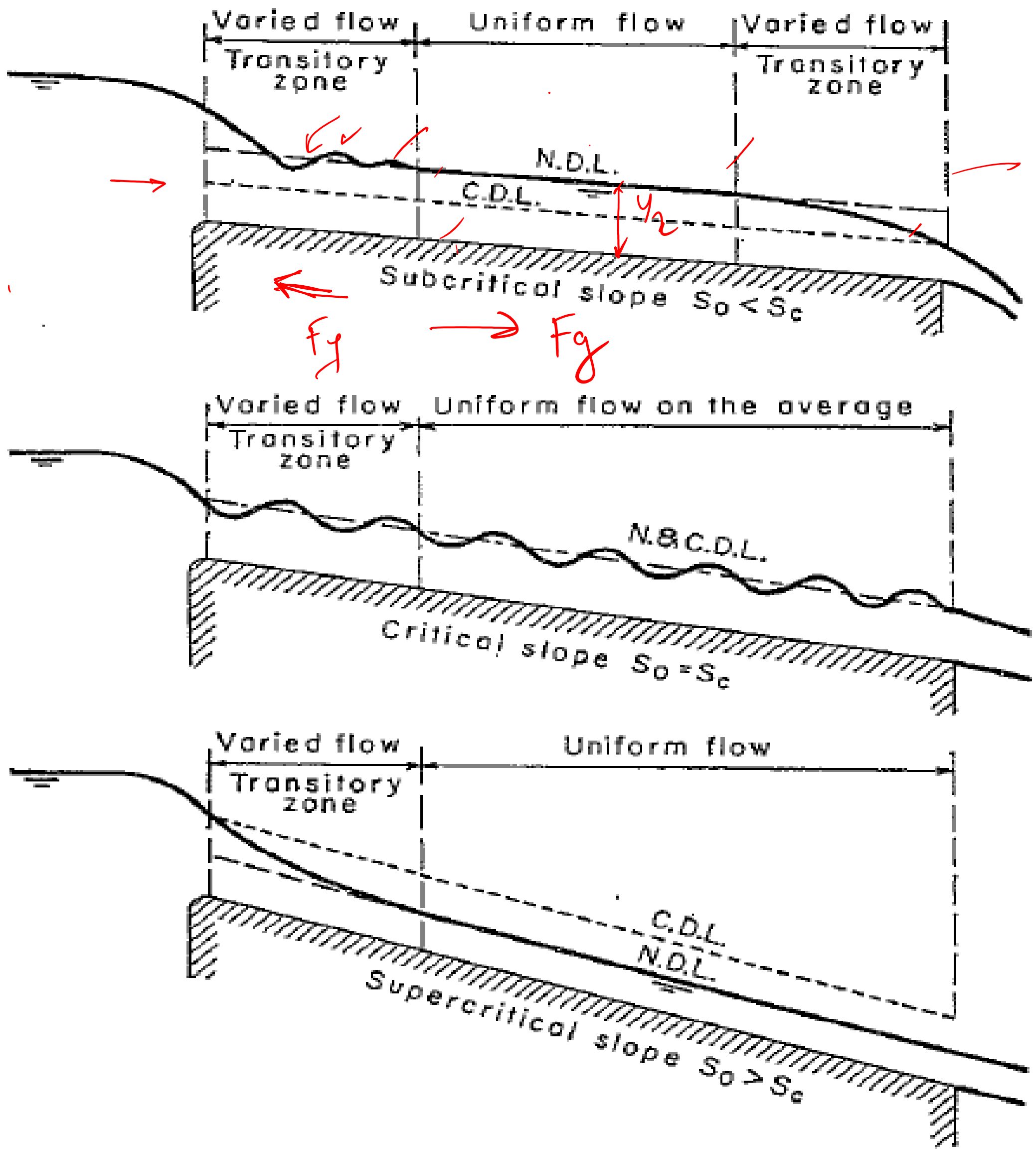


Fig.: Establishment of Uniform flow in long channel

Chapter 2

1. X-Section prop

2. Flow resistance eq^n

$\hookrightarrow Q = \frac{1}{n} A R^{2/3} S^{1/2}$

3. R varies y for ⊖ & ⊟

4. n variation

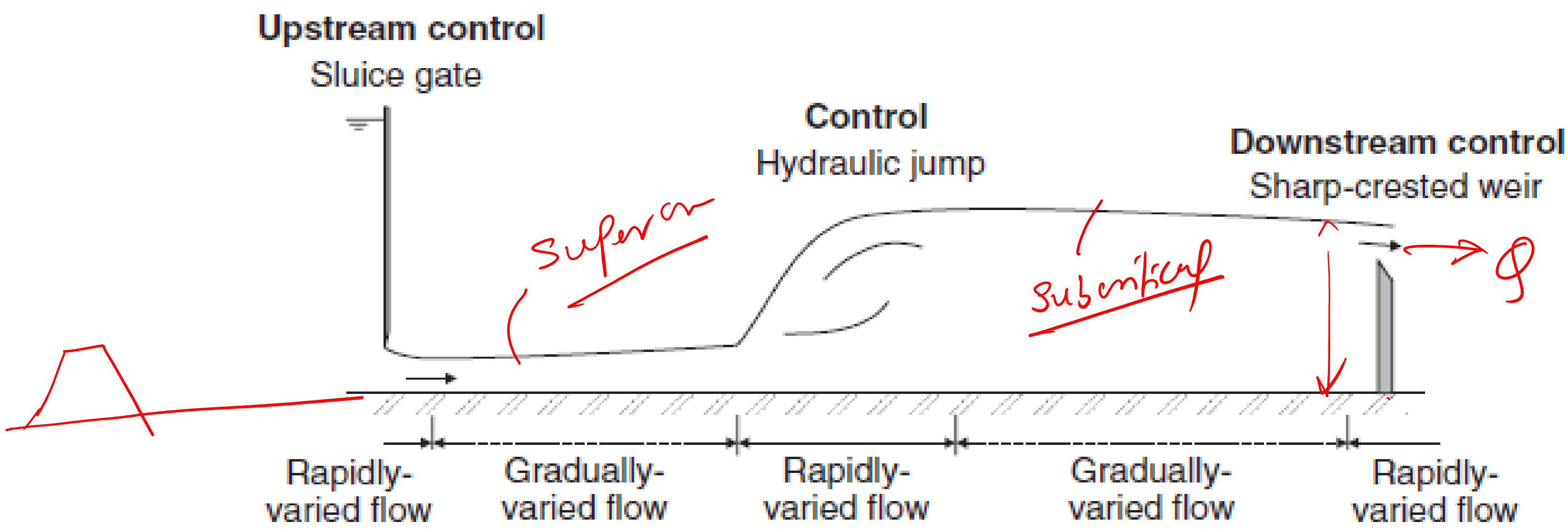
5. P, V, τ_0 distribution

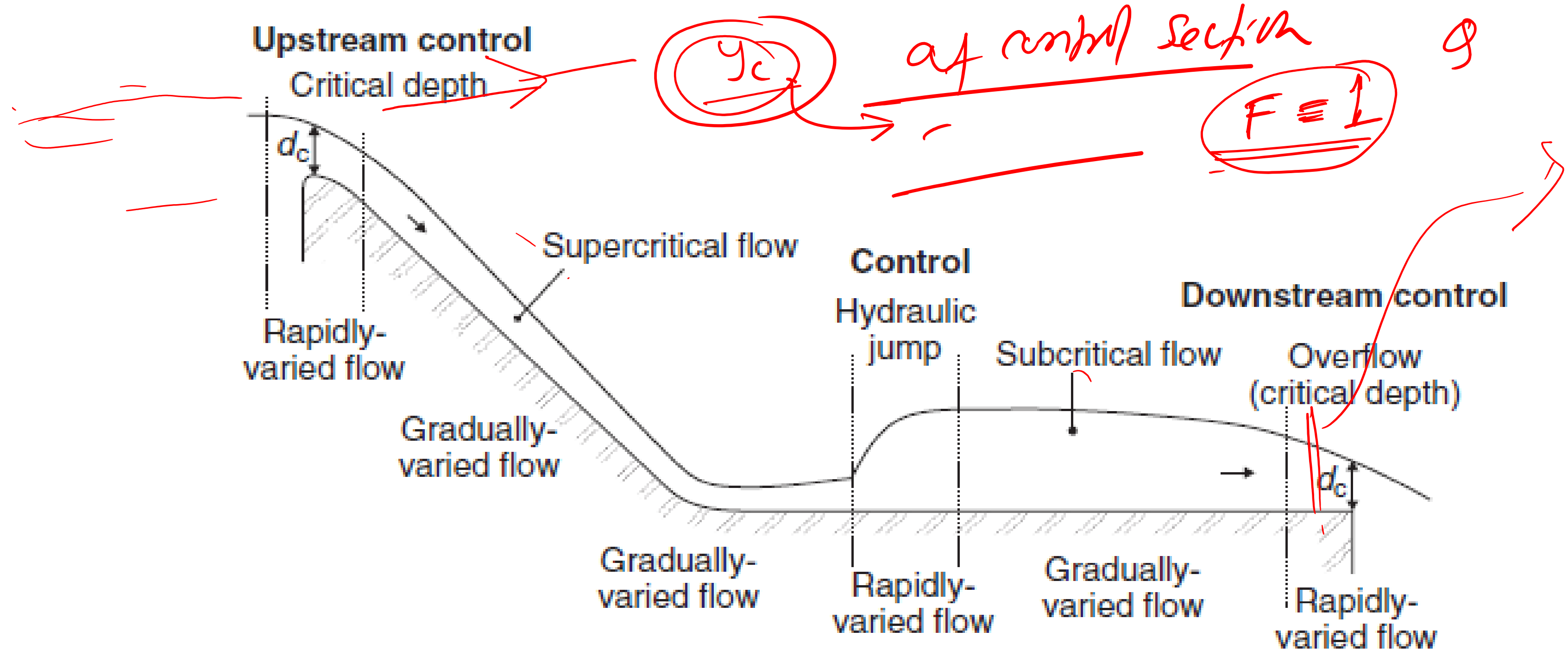
- **Normal Depth**
- **Critical depth**
- **Mild slope, Critical slope and Steep slope**

Control Section

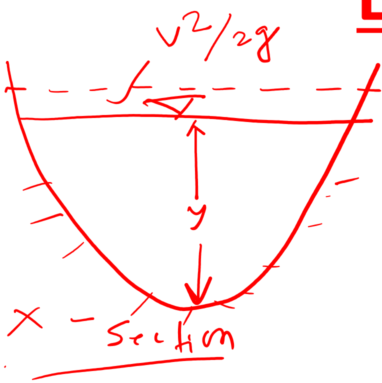
Whenever we drop a stone/or splash hand in flowing stream , there will be formation of wave and it will travel either at downstream or us/ds both. The wave which travels in downstream only then such flow is called supercritical flow. Since velocity of wave C is lower than stream velocity. And such flow is controlled at upstream. i.e., it is able to regulate supercritical flow at upstream control section. For example sluice gate is a control structure which regulates flow at downstream and a certain depth and discharge relationship can be established at such section.

Similarly , when wave travels at upstream, then such flow is subcritical flow. i.e., celerity is higher than flow velocity. It is easy to control subcritical flow by placing structures at downstream end. i.e., to control upstream section of flow, we have to put structure at downstream section.
For example. Spillway , weir are control sections for upstream flow control. And there exist unique relationship for depth and discharge at these structures

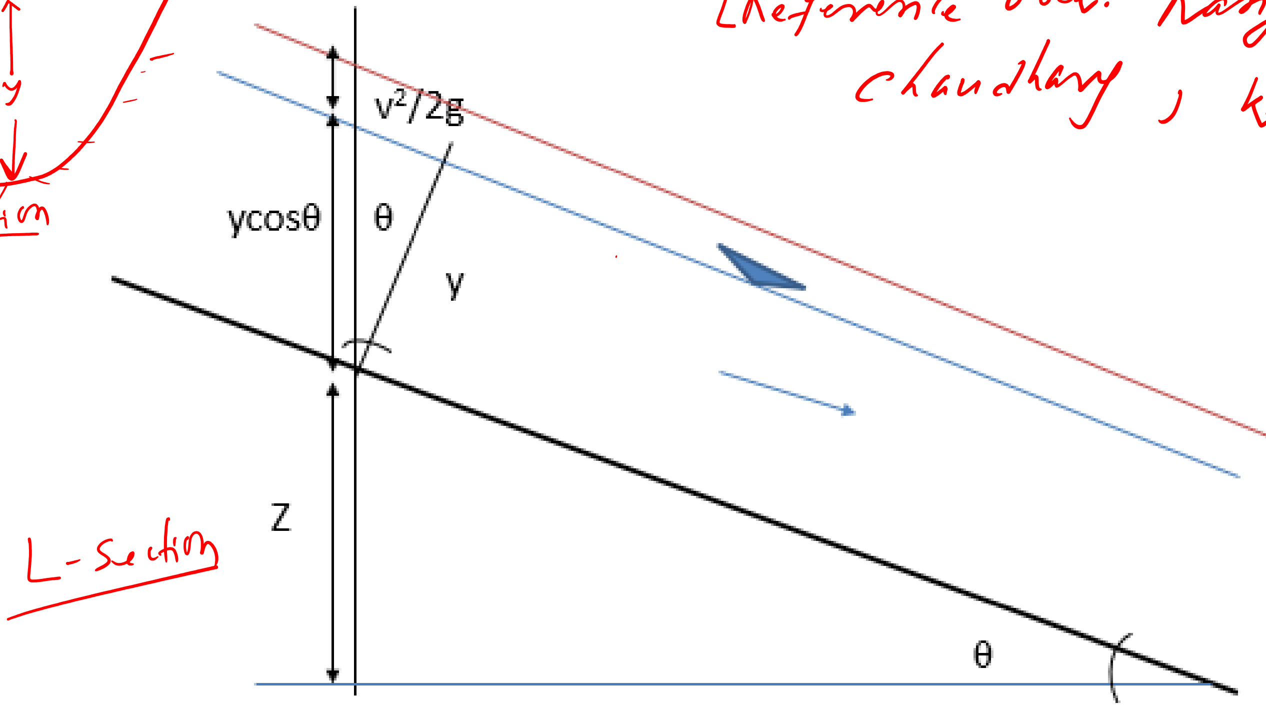

Upstream control
Sluice gate
Control
Hydraulic jump
Downstream control
Sharp-crested weir
Rapidly-varied flow
Gradually-varied flow
Rapidly-varied flow
Gradually-varied flow
Rapidly-varied flow


Upstream control
Critical depth
d_c
Rapidly-varied flow
Supercritical flow
Gradually-varied flow
Control
Hydraulic jump
Subcritical flow
Downstream control
Overflow (critical depth)
d_c
Gradually-varied flow
Rapidly-varied flow
Gradually-varied flow
Rapidly-varied flow

Energy – Depth Relationship



[Reference book: Ranga Raju, Hanif Chaudhary, K. Subramanya]



$$H = Z + Y\cos\theta + \alpha v^2/2g \qquad \text{.................................(1)}$$

for very small angle cos θ =1 and taking kinetic energy correction factor α=1

$$H = Z + Y + v^2/2g$$

considering channel bed as datum, then Z=0

$$E = Y + v^2/2g \qquad \text{........................(2)}$$

Here, the term $E = y + \frac{V^2}{2g}$, called as <u>Specific Energy</u>, introduced by <u>Bakehmeteff</u> is very useful in defining <u>critical depth</u>

We have Specific energy, $E = y + V^2/2g = y + \frac{Q^2}{2g(By)^2}$

$$q = \frac{Q}{B}$$

$$E = y + \frac{q^2}{2gy^2}$$

or, $$E - y = \frac{q^2}{2gy^2}$$

or, $$(E-y)y^2 = \frac{q^2}{2g} = \text{constant}$$

Mathematically, we can prove that E-y curve has two asymptotes [Asymptotes is a line that a curve approaches, as it heads towards infinity.]

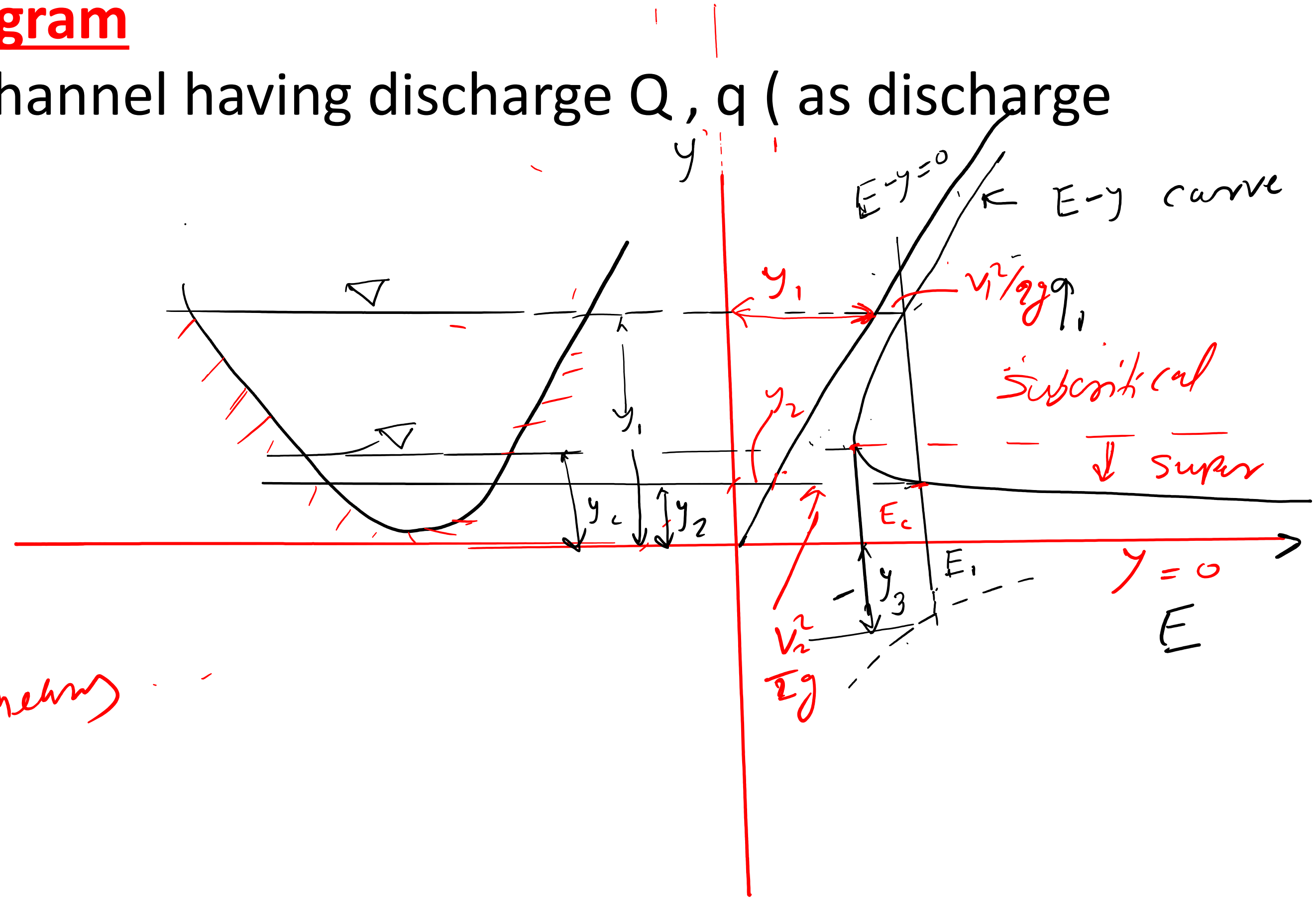
Energy –depth diagram

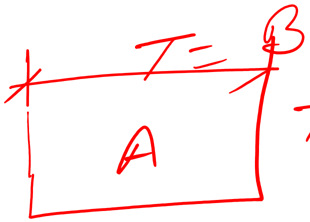
let us consider a channel having discharge Q , q (as discharge per unit width).

$$E = y + \frac{V^2}{2g}$$

$$q_1 = E_1$$

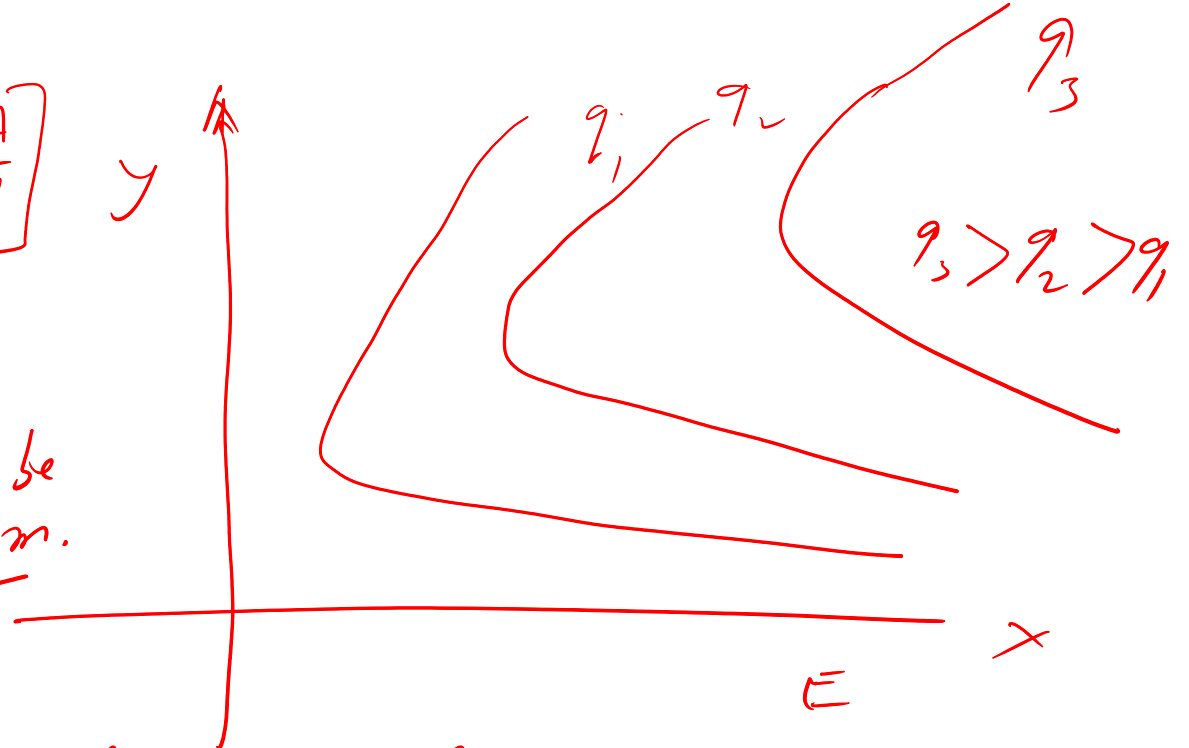
If means





$T = \frac{A}{y}$

at critical depth Specific Energy will be minm.



$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

$$\left(\frac{dA}{dy} = T\right)$$

for minm. E,

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 + \frac{Q^2}{2g}\left(\frac{dA^{-2}}{dy}\right)$$

$$= 1 - \frac{Q^2}{2g}\left(-2A^{-3}\right)\frac{dA}{dy}$$

$$= 1 - \frac{Q^2 T}{gA^3}$$

$$\frac{dE}{dy} = 0 = 1 - \frac{q^2 T}{g A^3} \Rightarrow \boxed{\frac{q^2 T}{g A^3} = 1}$$ — critical flow condn

$$\Rightarrow \frac{A^2 V^2 T}{g A^3} = 1$$

$$\frac{A}{T} = y$$

$$\Rightarrow \frac{V^2 \times T}{g A} = 1$$

$$F^2 = \frac{V^2}{g y}$$

$$\Rightarrow \frac{V^2}{g y} = 1 \Rightarrow F_r^2 = 1$$

$$\boxed{F = 1}$$

$$\frac{V^2}{g y_c} = 1$$

$$\boxed{y_c = ?}$$

$$\boxed{y_c = \left(q^2/g\right)^{1/3}}$$

$$E = y_c + V^2/2g = y_c + \frac{y_c}{2}$$

$$\frac{V^2}{2g} = \frac{y_c}{2}$$

$$\boxed{E_{min} = \frac{3}{2} y_c}$$

at critical flow, $F = 1$

$$\frac{V}{\sqrt{g y_c}} = 1$$

$$\frac{q}{B} = q$$

or $\frac{V^2}{g y_c} = 1 \Rightarrow \frac{Q^2}{g y_c * A_c^2} = 1$

or $\frac{Q^2}{g y_c * B^2 y_c^2} = 1$

or $\frac{q^2}{g y_c^3} = 1 \Rightarrow y_c^3 = \frac{q^2}{g}$

$$y_c = \left(q^2 / g \right)^{1/3}$$

Dimensionless Specific Energy

$$E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2}$$

$$\left\{ y_c = \left(\frac{q^2}{g}\right)^{1/3} \right.$$

or) $$\frac{E}{y_c} = \frac{y}{y_c} + \frac{y_c^3}{2y^2\, y_c}$$

$$y_c^3 = \frac{q^2}{g}$$

$$\frac{E}{y_c} = \frac{y}{y_c} + \frac{y_c^2}{2y^2} = f\left(\frac{y}{y_c}\right)$$

$$\frac{E}{y_c} = f\left(\frac{y}{y_c}\right)$$

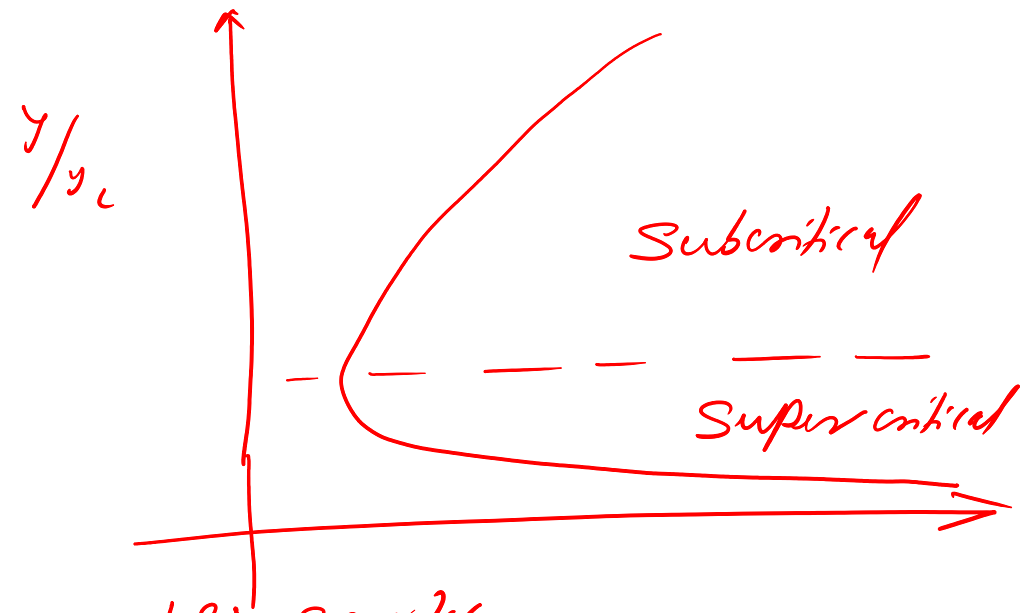


Fig: Dimensionless specific energy curve E/yc

Depth – Discharge diagram

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2} \quad \text{--- (I)}$$

or) $$E - y = \frac{Q^2}{2gA^2}$$

or) $$2gA^2 \times (E - y) = Q^2$$

or) $$Q = A\sqrt{2g}\sqrt{E - y} \quad \text{--- (II)}$$

in eqn (II): if $y = 0$, $A = 0$, $Q = 0$

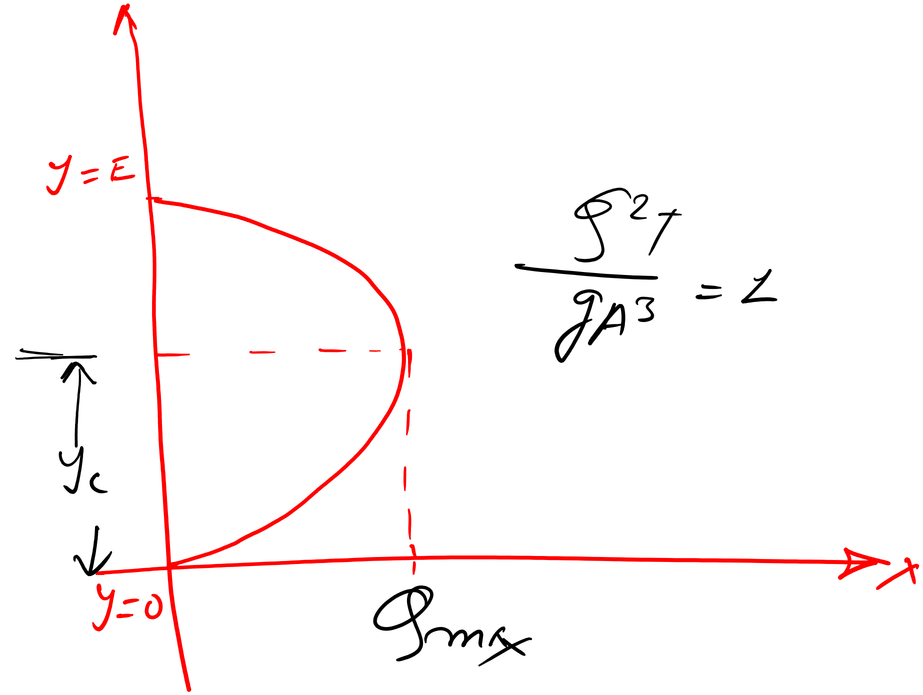
at $y = E$, $Q = 0$

$$Q = A\sqrt{2g}\,\sqrt{E-y}$$

at $y = 0$; $A = 0$, $Q = 0$

at $y = E$, $Q = 0$

for Q_{max} , $\frac{dQ}{dy} = 0$



$$\frac{Q^2 T}{gA^3} = 1$$

$$\frac{dQ}{dy} = \sqrt{2g}\,\frac{d\left(A\sqrt{E-y}\right)}{dy}$$

$$= \sqrt{2g}\left[A\,\frac{d\sqrt{E-y}}{dy} + \sqrt{E-y}\,\frac{dA}{dy}\right]$$

$\frac{dA}{dy} \rightarrow T$

$$\frac{dQ}{dy} = 0 ,$$

$$E - y = \frac{A}{2T} \quad \text{— (11)}$$

$$E = y + \frac{Q^2}{2gA^2}$$

or $$E - y = \frac{Q^2}{2gA^2}$$

from eqn (iii) $E - y = A/2T$, in above eqn,

or $$\frac{A}{2T} = \frac{Q^2}{2gA^2}$$

or $$\boxed{\frac{Q^2 T}{gA^3} = 1} \Rightarrow Fr = 1$$

Thus a channel can pass a maxm. discharge at critical flow maintaining constant Sp. Energy

$E = 5m$

$$5 = y + \frac{V^2}{2g} = y + \frac{q^2}{2g B^2 y^2}$$

$$5 - y = \frac{q^2}{2g B^2 y^2}$$

$$(5-y)\,(2g B^2)\,y^2 = q^2$$

$$\frac{q^2}{gA^3} = 1$$

let $y = 3$

$5 = 0$

$y = 3 \rightarrow 2 \times 3^2 = 18$ → y_c →

$y = 4 \rightarrow 1 \times 4^2 = 16$ ✓

$y = 2 \rightarrow 3 \times 2^2 = 12$ ✓

Control Section