lateral inflow

Fig: Side-channel spillway as an example for spatially varied flow

<u>Up to now we have studied following topics</u>:

- Condition of uniform flow, expression for the shear stress on boundary of channel.
- Flow resistance equation
- Determination and factors affecting Manning's n
- Velocity distribution, shear stress distribution, Pressure distribution
- Energy and momentum correction factor

- Conveyance, Section factor, normal depth, critical depth
- Hydraulic Efficient channel sections (Rectangular, Trapezoidal, circular and triangular)
- Hydraulic exponent of channels

Conveyance: It is a carrying capacity of channel.

For uniform flow Q= $\frac{1}{n}AR^{2/3}S^{1/2}$, where S is slope of energy lines , but for uniform flow , it is equals to bed slope.

$Z = A\sqrt{\frac{A}{T}}$

Conveyance k = $\frac{1}{n}AR^{2/3}$

For uniform flow, section factor Z may be written as = $AR^{2/3}$,

Generally for trapezoidal and rectangular , circular channel , it becomes tedious for computation of normal depth of channel for given Q, n and S. We have to use trial and error method for computation of it or Newton Raphson method.

So, a Section factor vs normal depth curve should be developed for the determination of normal depth for given Q,n, and S.

Z vs y curve,

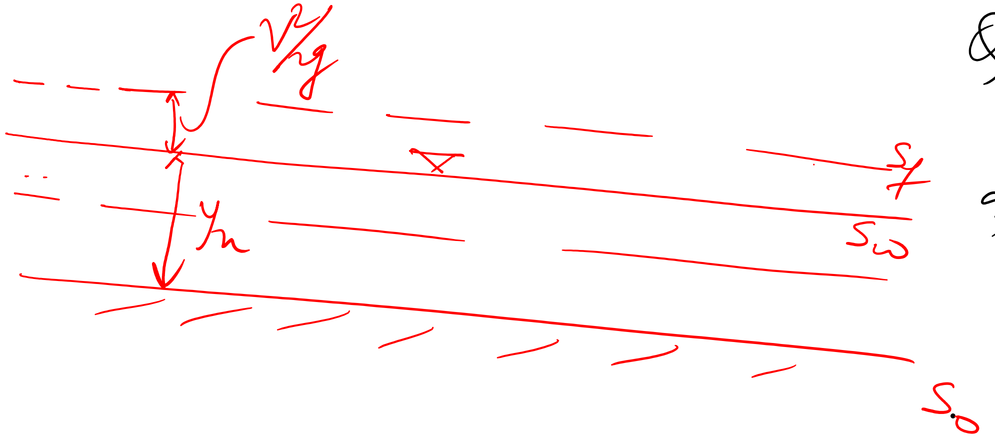
Measure X-section and

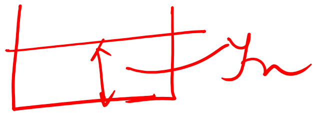
$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

or

$$\boxed{\frac{nQ}{\sqrt{S}} = Z}$$

$$Q = \frac{1}{n} A R^{2/3} S_f^{1/2}$$





$$A = b y_n$$

$$R^{2/3} = A/P = \left(\frac{b y_n}{b + 2 y_n}\right)^{2/3}$$

$$S_f = S_w = S_o$$

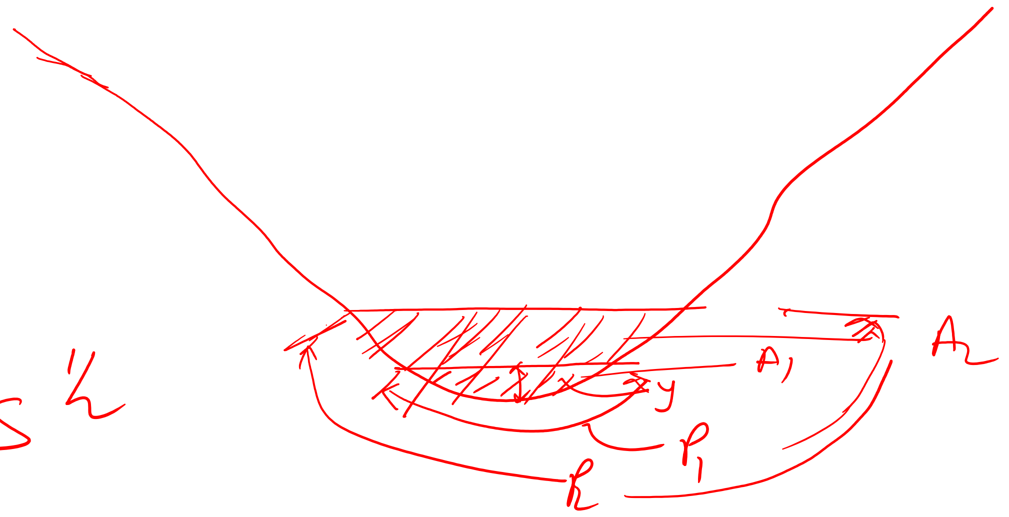
of uniform flow

y_n

Trial & error

Irregular channel

$Q \to y_n = ?$



$Z = A R^{2/3}$

$= A \times \left(\frac{A}{P}\right)^{2/3}$

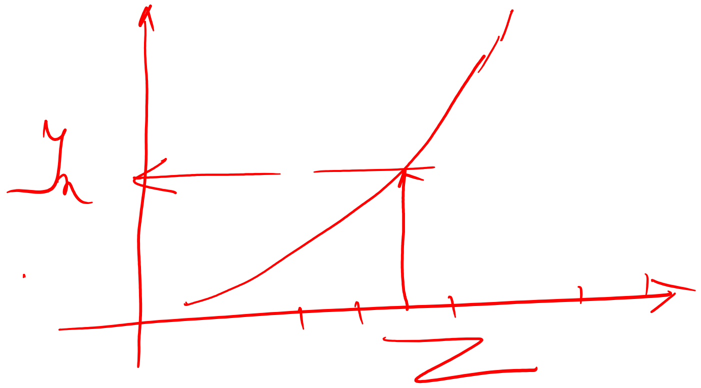
$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$\Rightarrow Q = \frac{1}{n} Z S^{1/2}$

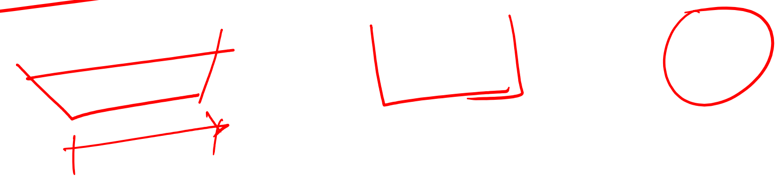
$\Rightarrow Z = \frac{Q n}{\sqrt{S}}$

y	Z
1	—
2	—
3	—
4	—

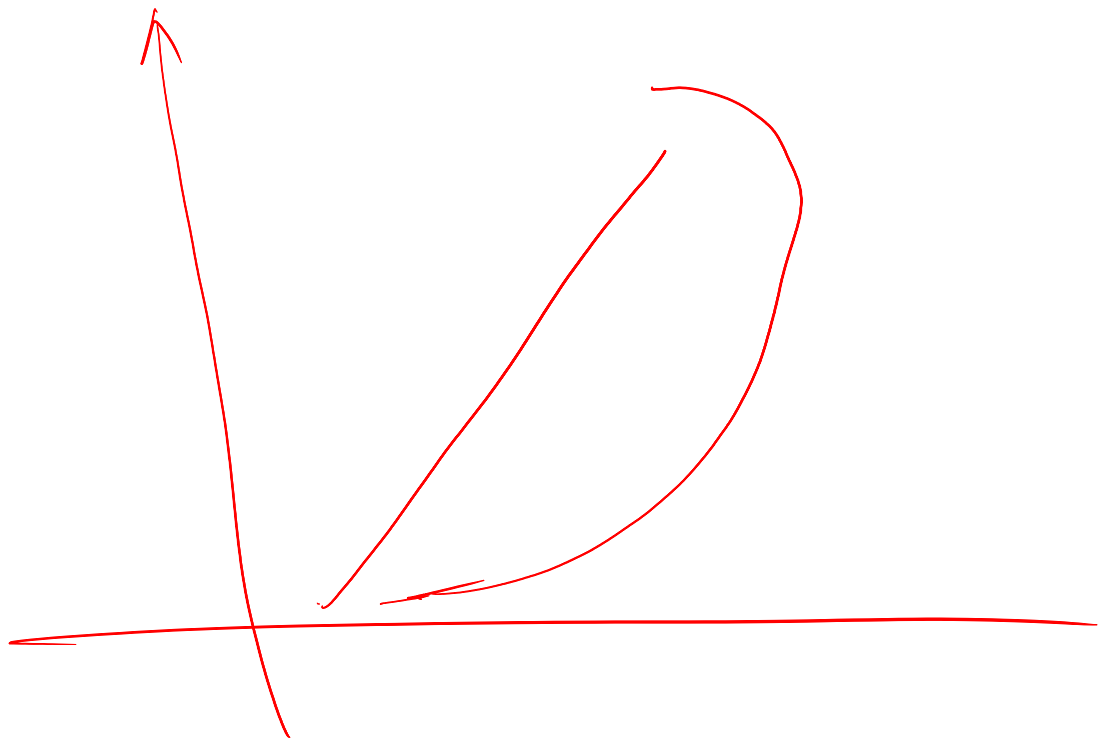
Q, n, S , $Z = \boxed{\ } \to y_n$



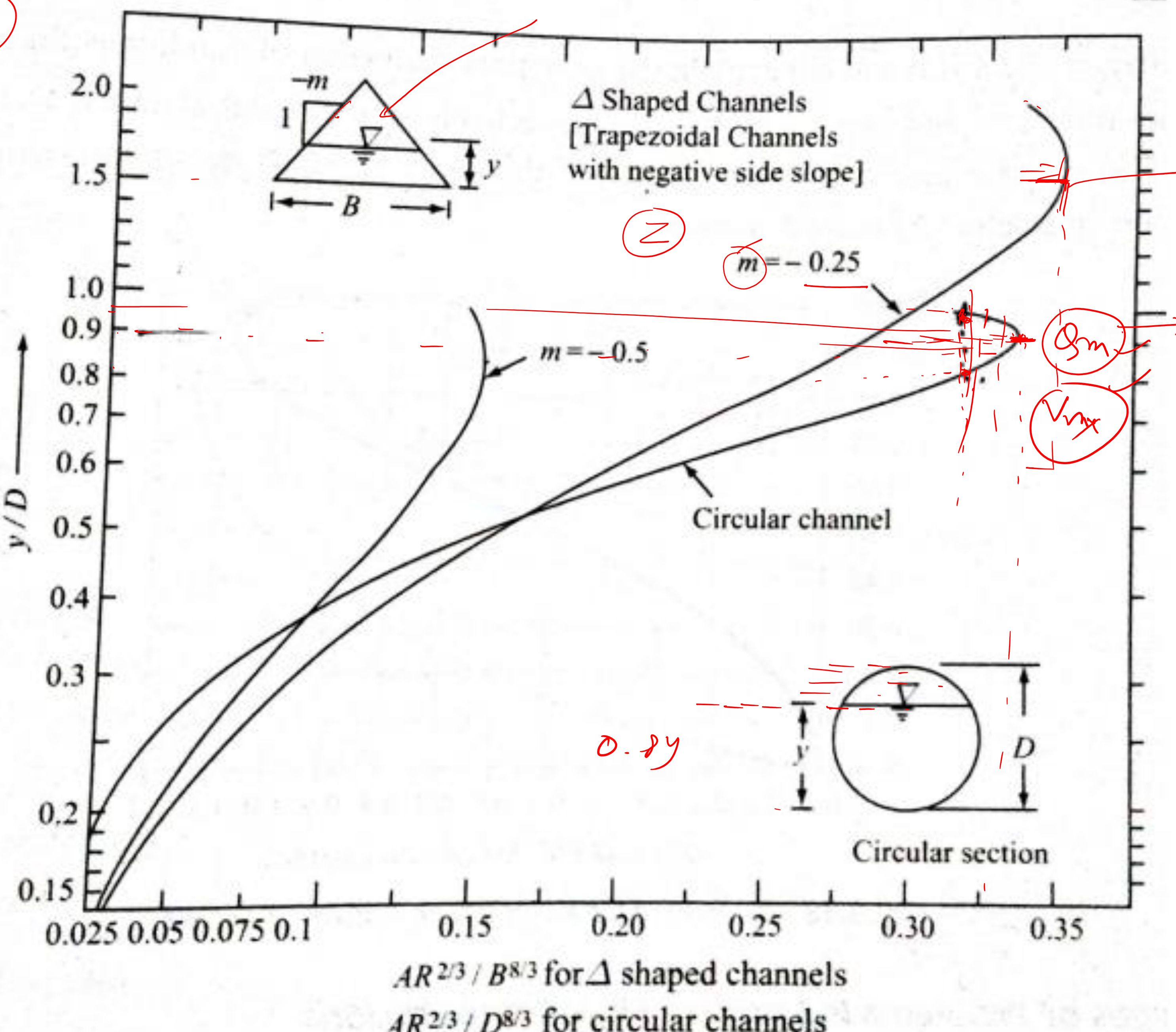
Regular



y/B vs Z

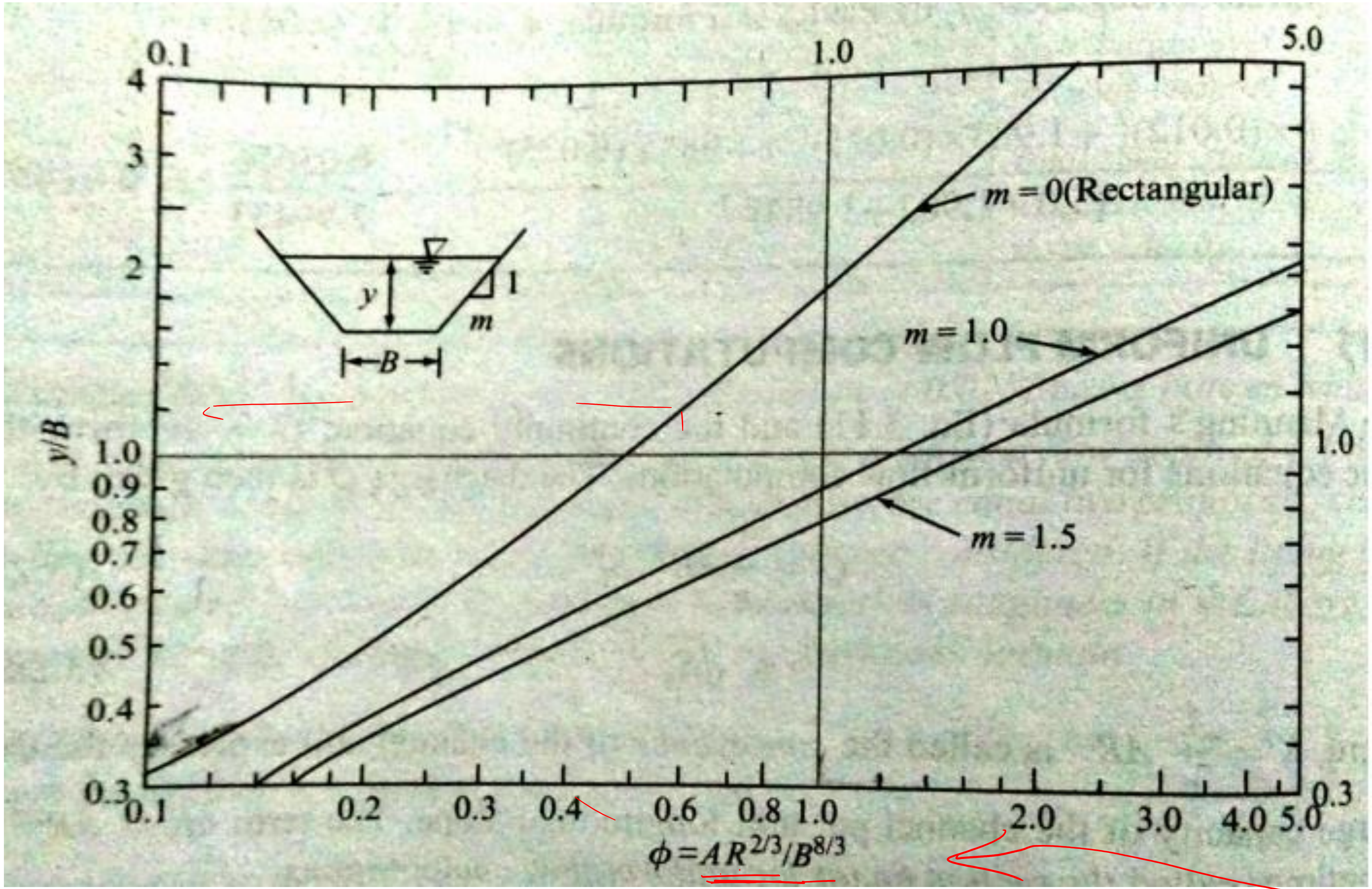


z:1



$Z = \frac{nQ}{\sqrt{S}}$

for same Q
these channels
have two
normal depth



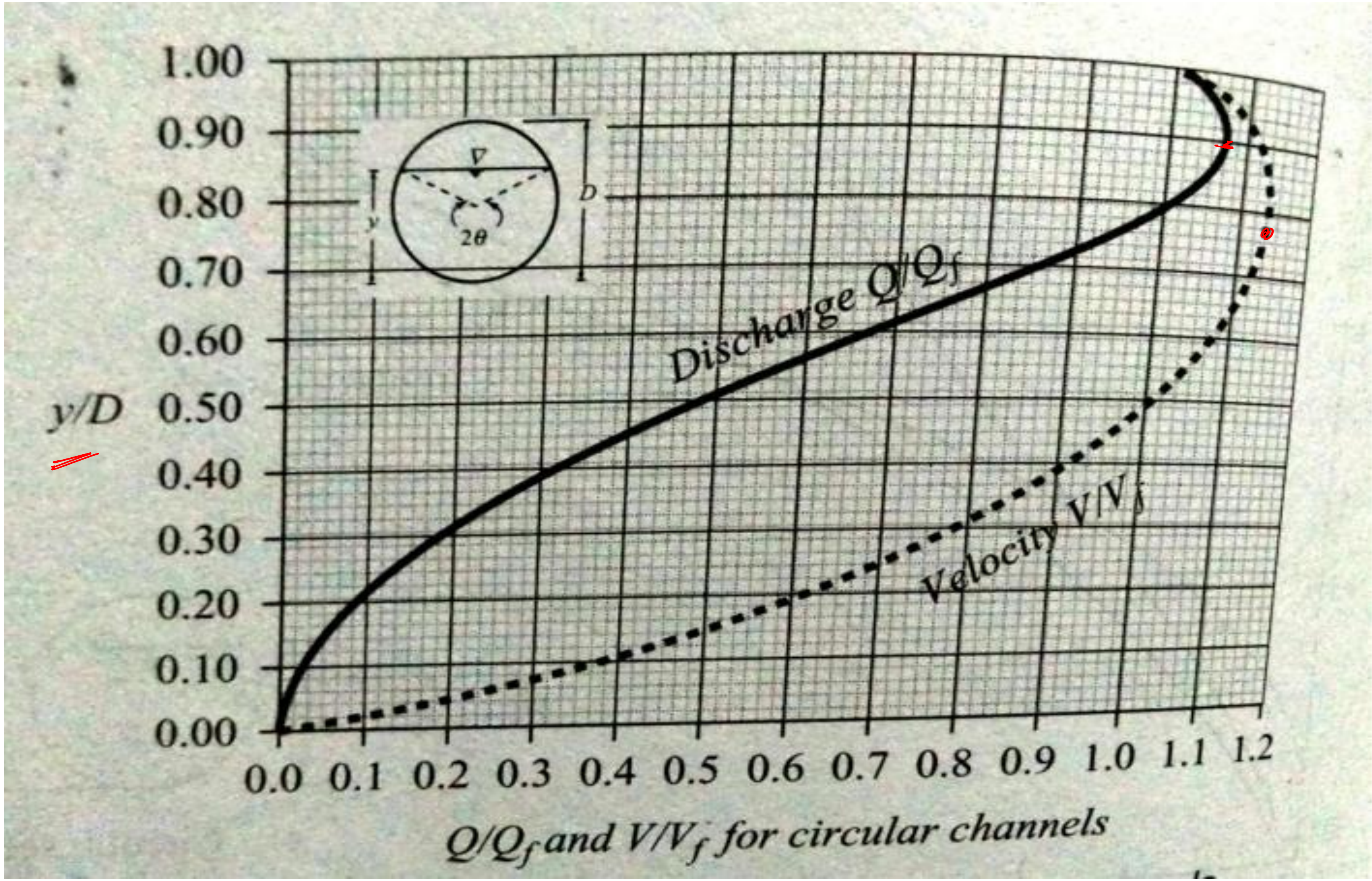
(2) $\frac{AR^{2/3}}{B^{8/3}}$

$y/B = 1$

$\boxed{y_n = 1 \times B}$

$Z = \frac{Qn}{\sqrt{S}}$

$Z/B^{8/3}$



V_f

Q_f

either

$Q_f < Q_{max}$

Maximum discharge and maximum velocity in a channel of a second kind. (closed conduit)

Channels of the second kind have two normal depths in a certain range of depth and there exists a finite depth at which these sections have a finite depth at which the velocity of flow is maximum.

Maximum discharge

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Z = A R^{2/3} \propto = A \times \left(\frac{A}{P}\right)^{2/3} \rightarrow \text{maxm}$$

~~minm~~

$$Q = \frac{\sqrt{S}}{n} \times A \times \left(\frac{A}{P}\right)^{2/3}$$

$$= \frac{\sqrt{S}}{n} \frac{A^{5/3}}{P^{2/3}} = \frac{\sqrt{S}}{n} \left(\frac{A^5}{P^2}\right)^{1/3}$$

$$\frac{dp}{dy} = 0$$

$$\Rightarrow \quad \frac{d\left(\frac{A^5}{p^2}\right)}{dy} = 0$$

$$\Rightarrow \quad \frac{p^2 \frac{dA^5}{dy} - A^5 \frac{dp^2}{dy}}{(p^2)^2} = 0$$

$$y_n =$$

$\Rightarrow$

for velocity

$$\boxed{\cancel{V = C\sqrt{RS}}}$$

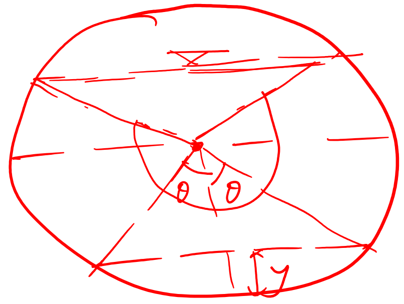
$$V = \frac{1}{\eta} R^{2/3} S^{1/2}$$

$$= \left(\frac{A^2}{P^2}\right)^{1/3}$$

$$\frac{dV}{dy} = 0$$

$$\frac{d\left(\frac{A^2}{P^2}\right)}{dy} = 0 \qquad \Rightarrow y = \square\ \text{le}$$

Q Analyse the condns reqd to have (a) maxm. discharge and (b) maxm. velocity of circular channel.



$A = \frac{D^2}{8}(2\theta - \sin 2\theta)$

$P = D\theta$

$Q = \frac{1}{n}(A R^{2/3} S^{1/2})$

$\frac{dQ}{d\theta} = \frac{d}{d\theta}\left(\frac{A^5}{P^2}\right) = 5P\frac{dA}{d\theta} - 2A\frac{dP}{d\theta} = 0$

$\Rightarrow 5D\theta \frac{d}{d\theta}\frac{D^2}{8}(2\theta - \sin 2\theta) - 2 \times \frac{D^2}{8}(2\theta - \sin 2\theta)\frac{d(D\theta)}{d\theta} = 0$

$\Rightarrow 3\theta - 5\theta\cos 2\theta + \sin 2\theta = 0$

solving this eqn $\theta = 150^\circ\ 11'$ <

$$\cos\theta = \left(1 - \frac{2y}{D}\right)$$

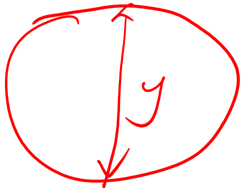
$$\Rightarrow \frac{y}{D} = \frac{1-\cos\theta}{2} = 0.938$$

$$\boxed{y = 0.938D} \longrightarrow Q = Q_{max}$$

at $y/D = 0.938$

$$\frac{AR^{2/3}}{D^{8/3}} = 0.3353 \rightarrow Q_m \ (\text{at } \theta = 150°11'')$$

at $\frac{y}{D} = 1.0$



$$\frac{AR^{2/3}}{D^{8/3}} = 0.3117 \ (\text{at } \theta = 360°) \rightarrow Q_f$$

$$\frac{Q_{max}}{Q_f} = \frac{0.3353}{0.3117} = 1.0757$$

$$\left(\frac{Q_m - Q_f}{Q_f}\right)100 = 7.6\%$$

Maximum velocity

$V = \frac{1}{n} R^{2/3} S^{1/2}$

$= \left(\frac{A^2}{P^2}\right)^{1/3}$

$\frac{dV}{d\theta} = 0$

$\frac{d}{d\theta}\left(\frac{A^2}{P^2}\right) = 0$

$2\theta = \boxed{}$ $\underline{\underline{257.45^\circ}}$

$\theta = \boxed{}$

y/D

$\frac{y}{D} = \frac{1 - \cos\theta}{2} \Rightarrow y/D = 0.8128$

Chezy's eqn

$\boxed{y_0 = 0.95 D}$

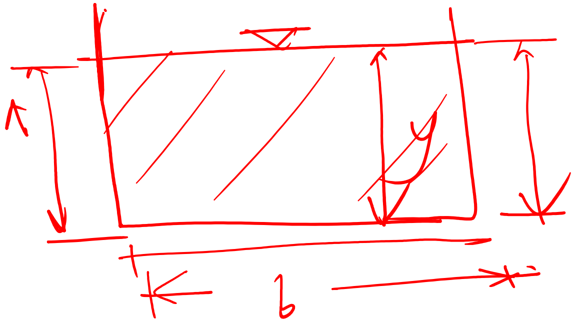
Hydraulic Efficient Channels

$A = by$

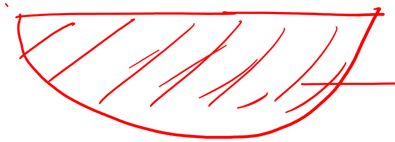
$P = b + 2y$

$R = A/P$

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$



$y \rightarrow \quad A \rightarrow$



$A = \frac{\left(\frac{\pi d^2}{4}\right)}{2}$

$P = \left(\frac{A}{y} + 2y\right)$

if P is to be minm. with A const

$\frac{dP}{dy} = 0$, or

$-\frac{A}{y^2} + 2 = 0$

$\Rightarrow A = 2y^2$

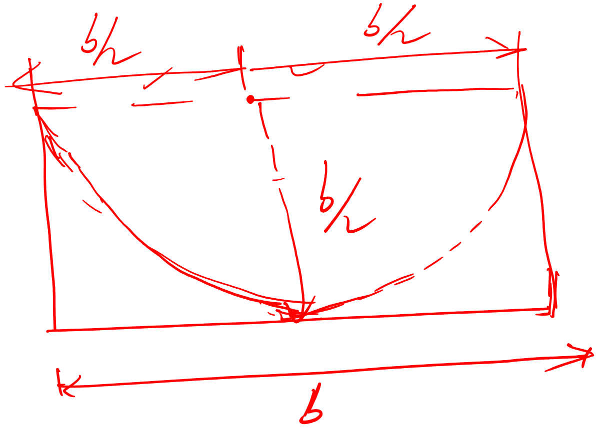
$by = 2y^2 \Rightarrow \boxed{y_0 = b/2}$

$y_c = b/2$

$R = A/P$

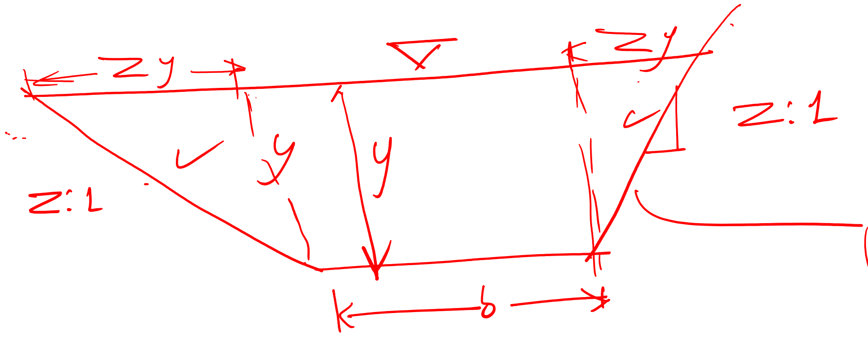
$= \frac{by}{b+2y} = \frac{2y \cdot y}{2y+2y}$

$\boxed{R = y_c/2}$



$y_c = b/2$

Trapezoidal Channel



$\sqrt{(Zy)^2+y^2} = y\sqrt{Z^2+1}$

$A = by + Zy^2$

$P = b + 2*y\sqrt{Z^2+1}$

$= \frac{A - Zy^2}{y} + 2y\sqrt{Z^2+1}$

In order to minimise P, we have $\frac{dP}{dy} = 0$

θ. $P = \frac{A}{y_e} - zy_e + 2y_e\sqrt{z^2+1}$

$$\frac{dP}{dy} = \frac{-A}{y_e^2} - z + 2(\sqrt{z^2+1}) = 0$$

$$A = (2\sqrt{z^2+1} - z)\, y_e^2$$

$$B_e = 2y(\sqrt{z^2+1} - z)$$

$$P_e = 2y(2\sqrt{1+z^2} - z)$$

$$R_e = \frac{A}{P} = y_e/2$$

Hydraulic exponent of channels (N)

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = \frac{\sqrt{S}}{n} A R^{2/3} \text{ — } Z$$

$$K = \frac{1}{n} A R^{2/3} \text{ — } f(y^2)$$

$$\boxed{K = C y^N}$$

$$\boxed{\Rightarrow \quad K^2 = C^2 y^N}$$

Bakhmeteff

Integral

$$\boxed{K^2 = C_2 y^N}$$

An appx expression for N

$$\boxed{k^2 = c_2\, y^{N}}$$

$$k = \frac{1}{\eta} A R^{2/3}$$

$$\therefore\ k^2 = \frac{1}{\eta^2} A^2 R^{4/3}$$

$$= \frac{1}{\eta} A^2 \left(\frac{A}{P}\right)^{4/3} = \frac{1}{\eta^2} A^{10/3} P^{-4/3} = c_2\, y^{N}$$

$$\Rightarrow\ \frac{1}{\eta^2} A^{10/3} P^{-4/3} = c_2\, y^{N}$$

taking log both side we h

$$\ln\left(\frac{1}{\eta^2}\right) + \frac{10}{3}\ln A - \frac{4}{3}\ln P = \ln C_2 + N \ln y$$

Differentiating both side w.r.t y

$$\frac{10}{3}\,\frac{dA/dy}{A} - \frac{y}{3}\,\frac{dP/dy}{P} = \frac{N}{y}$$

$$\boxed{T = \frac{dA}{dy}} \qquad \boxed{y = \frac{A}{T}}$$

$$\Rightarrow N = \frac{2y}{3}\left(\frac{5T}{A} - \frac{2\,\frac{dP}{dy}}{P}\right)$$

Q obtain the value of N for (a) wide rectangular channel and a (b) triangular channel

(a) $R = y$

$$R = A/P = \frac{by}{(b+2y)} \qquad (b+2y) = b \qquad \frac{by}{b} = y$$

$b \gg y$

$$\boxed{R = y}$$

② considering unit width of channel

$$A = 1 \times y = y$$

$$R = y$$

$$K^2 = \frac{1}{n^2} y^2 \cdot y^{4/3} = C_2 y^N$$

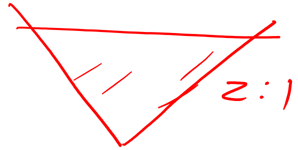
$$\frac{1}{n^2} y^{2+4/3} = C_2 y^N$$

$$\frac{1}{n} y^{10/3} = C_2 y^N$$

(i) taking log both side

(ii) differentiating

$$N = \frac{10}{3} = 3.33$$



Similarly for triangular channel

$$N = 5.33$$

Hydraulic efficient channel section

$$\boxed{Q = \frac{1}{n} A R^{2/3} S^{1/2}}$$

$$K = \frac{1}{n} A R^{2/3} \quad , \quad \uparrow R = A/P \downarrow$$

1) Rectangular channel.
2) Trapezoidal channel
3) Triangular channel.

- **Hydraulic exponent of channels**